Clustering Analysis Method based on Fuzzy C-Means Algorithm of PSO and PPSO with Application in Image Data

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Abstract: The popular fuzzy c-means algorithm (FCM) converges to a local minimum of the objective function. Hence, different initializations may lead to different results. The important issue is how to avoid getting a bad local minimum value to improve the cluster accuracy. The particle swarm optimization (PSO) is a popular and robust strategy for optimization problems. But the main difficulty in applying PSO to real-world applications is that PSO usually need a large number of fitness evaluations before a satisfying result can be obtained. In this paper, the improved new algorithm, “Fuzzy C-Mean based on Picard iteration and PSO (PPSO-FCM)”, is proposed. Two real data sets were applied to prove that the performance of the PPSO-FCM algorithm is better than the conventional FCM algorithm and the PSO-FCM algorithm.

Key-Words: FCM, Picard iteration, PSO algorithms, PPSO-FCM algorithm.

1 Introduction
The popular fuzzy c-means algorithm (FCM) is developed by using Picard Iteration through the first-order conditions for stationary points of the objective function. It converges to a local minimum of the objective function. Hence, different initializations may lead to different results. The important issue is how to avoid getting a bad local minimum value to improve the cluster accuracy.

The particle swarm optimization (PSO) is a popular and robust strategy for optimization problems. But the main difficulty in applying PSO to real-world applications is that PSO usually need a large number of fitness evaluations before a satisfying result can be obtained. In this paper, the improved new algorithm, “Fuzzy C-Mean based on Picard iteration and PSO (PPSO-FCM)”, is proposed. Two real data sets were applied to prove that the performance of the PPSO-FCM algorithm is better than the conventional FCM algorithm and PSO-FCM algorithm.

2 Literature Review
The algorithm of fuzzy C-Means Algorithm are the foundations of this study[1][2][3]. The algorithm will be discussed as follows.

2.1 Fuzzy c-Mean Algorithm

Fuzzy C-Means Algorithm (FCM) is the most popular objective function based fuzzy clustering algorithm. The objective function used in FCM is given by Equation (1)

\[ J_{FCM}^n(U,A,X) = \sum_{i=1}^{c} \sum_{j=1}^{n} \mu_{ij}^m d_{ij}^2 = \sum_{i=1}^{c} \sum_{j=1}^{n} \mu_{ij}^m \| x_j - a_i \| \]  

\[ \mu_{ij} \in [0,1] \] is the membership degree of data object \( x_j \) in cluster \( C_i \) and it satisfies the following constraint given by Equation (2).

\[ \sum_{i=1}^{c} \mu_{ij} = 1, \forall j = 1,2,...,n \]  

C is the number of clusters, \( m \) is the fuzzy index, \( m>1 \) (in this paper \( m=2 \)), which controls the fuzziness of the method. They are both parameters and need to be specified before running the algorithm. \( d_{ij}^2 = \| x_j - a_i \| \) is the square Euclidean distance between data object \( x_j \) to center \( a_i \).

Minimizing objective function Equation (1) with constraint Equation (2) is a non-trivial constraint nonlinear optimization problem with continuous parameters \( a_i \) and \( \mu_{ij} \). Hence there is no obvious analytical solution. Alternatively optimizing one set of parameters while the other set of parameters are considered as fixed, is used here. The updating function for \( a_i \) and \( x_j \) is obtained as Equation (3) and (4).

The steps of the FCM are listed as follows.

Step 1: Determining the number of cluster; \( c \) and \( m \)-value (let \( m=2 \)), given converging error, \( \varepsilon > 0 \) (such as \( \varepsilon = 0.001 \)), randomly choose the initial membership matrix, such that the memberships \( \mu_{ij}^{(0)} \), \( i = 1,2,...,c, j = 1,2,...,n \) are not all equal.

Step 2: Find

\[ a_i^{(k)} = \frac{\sum_{j=1}^{n} \mu_{ij}^{(k-1)} x_j}{\sum_{j=1}^{n} \mu_{ij}^{(k-1)}} \quad i = 1,2,...,c \]  

Step 3: Increment k; Until \( \max_{1 \leq i \leq c} \| a_i^{(k)} - a_i^{(k-1)} \| < \varepsilon \)  

2.2 PSO-FCM Algorithm

Particle Swarm Optimization (PSO)[7] is a quite convenient method for optimizing hard numerical function on metaphor of social behavior of flocks of birds and schools of fish[5].

A swarm consist \( M \) individuals, \( ( \text{here, } 3 \leq M \leq 20 ) \), called particles, which change their position over time. Each particle represents a potential solution to the problem of optimization[6]. In FCM, the problem of optimization is to minimize the value of the objective function. Let the particle \( k \) in a D-dimension space \( (D=nc) \) be represented as

(i) Let the particle \( k \) in a D-dimension space \( (D=nc) \) be represented as

\[ \mu_k = (\mu_{k1}, \mu_{k2}, \ldots, \mu_{kn}) \]

\[ = (\mu_{k11}, \mu_{k12}, \ldots, \mu_{k1n}, \mu_{k21}, \mu_{k22}, \ldots, \mu_{k2n}, \ldots, \mu_{kcn}, \mu_{kcn}) \]  

\[ k = 1,2,...,M \]  

(ii) Let the objective function of FCM be the fitness function as follows,

\[ J_{FCM}^n(U,A,X) = \sum_{i=1}^{c} \sum_{j=1}^{n} \mu_{ij}^m d_{ij}^2 \]

\[ = \sum_{i=1}^{c} \sum_{j=1}^{n} \mu_{ij}^m \| x_j - a_i \| ^2 \]  

where \( a_i = (\sum_{j=1}^{n} \mu_{ij}^m) ^{-1} \sum_{j=1}^{n} \mu_{ij}^m x_j, i = 1,2,...,c \)  

(iii) Let the particle \( k \) in a D-dimension space \( (D=nc) \) be represented as

\[ \mu_k = (\mu_{k1}, \mu_{k12}, \ldots, \mu_{kn}) \]

\[ = (\mu_{k11}, \mu_{k12}, \ldots, \mu_{k1n}, \mu_{k21}, \mu_{k22}, \ldots, \mu_{k2n}, \ldots, \mu_{kcn}, \mu_{kcn}) \]  

\[ k = 1,2,...,M \]
Let the objective function of FCM be the fitness function as follows,

\[ J_{FCM}(U, A, X) = \sum_{i=1}^{n} \sum_{j=1}^{m} \mu_{ij}^m d_{ij}^2 \]

\[ = \sum_{i=1}^{n} \sum_{j=1}^{m} \mu_{ij}^m ||x_j - a||^2 \]  \hspace{1cm} (10)

where

\[ a = \left( \sum_{j=1}^{m} \mu_{ij}^m \right)^{-1} \left( \sum_{j=1}^{m} \mu_{ij}^m x_j \right), \hspace{1cm} i = 1, 2, ..., c \]  \hspace{1cm} (11)

The best previous position which possesses the best fitness value of particle \( k \) was denoted by \( p_k = (p_{k1}, p_{k2}, ..., p_{kn}) \), which is also called \( p_{best} \). The index of the best \( p_{best} \) among all the particles is denoted by the symbol \( g \). We called that the best fitness value of the position \( p_k = (p_{k1}, p_{k2}, ..., p_{kn}) \) was also \( g_{best} \). The velocity for the particle \( k \) is represented as \( v_k = (v_{k1}, v_{k2}, ..., v_{kn}) \).

\[ p_{best} \] and \( g_{best} \), location for iteration \( t \) according to following two formulas,

\[ v_k(t+1) = w v_k(t) + c_1 r_1 (p_{best}(t) - \mu_k(t)) + c_2 r_2 (g_{best}(t) - \mu_k(t)) \]

\[ \mu_k(t+1) = \frac{\mu_k(t+1)}{\sum_{i=1}^{c} \mu_{ij}^m(t+1)} \]  \hspace{1cm} (12)

\[ \forall i = 1, 2, ..., c, j = 1, 2, ..., n, k = 1, 2, ..., m \]

where

\[ \mu_{ij}^m(t+1) = \frac{U_{ij}^m(t) - \min_{1 \leq j \leq n} U_{ij}^m(t)}{\max_{1 \leq j \leq n} U_{ij}^m(t) - \min_{1 \leq j \leq n} U_{ij}^m(t)} \]

\[ U_{ij}^m(t) = \mu_{ij}^m(t) + v_{ij}(t+1) \]  \hspace{1cm} (13)

\[ k = 1, 2, ..., m, i = 1, 2, ..., c, j = 1, 2, ..., n \]

Where \( w \) is the inertia coefficient which is a constant in the interval \([0,1]\), and can be adjusted in the direction of linear decrease, (In this paper \( w=0.75 \)); \( c_1 \) and \( c_2 \) are learning rates which are nonnegative constants (In this paper, \( c_1 = c_2 = 2 \)); \( r_1 \) and \( r_2 \) are generated randomly in the interval \([0,1]\).

The termination criterion for iterations is determined according to whether the maximum generation or a designated value of the fitness is reached. In this paper, the given converging error is \( \epsilon = 0.001 \)

\[ \max_{1 \leq i \leq c} ||y(t+1) - a_i(t)|| < \epsilon = 0.001 \]

where

\[ a_i(t+1) = \frac{\sum_{j=1}^{n} \mu_{ij}^m(t) x_j}{\sum_{j=1}^{n} \mu_{ij}^m(t)}, i = 1, 2, ..., c \]  \hspace{1cm} (14)

2.3 PPSO-FCM Algorithm

The main difficulty in applying PSO to real-world applications is that PSO usually need a large number of fitness evaluations before a satisfying result can be obtained[8]. For overcoming above problem, In this paper, the improved new algorithm, “Fuzzy C-Mean based on Picard iteration [4] and PSO (PPSO-FCM)”, is proposed. This algorithm integrates Picard iteration and PSO for FCM.

(i) Randomly choose the first particle in a D-dimensional space \( (D = nc) \) as follows, such that the memberships are not all equal

\[ \mu_i = (\mu_{i11}, \mu_{i12}, ..., \mu_{i1n}, \mu_{i21}, \mu_{i22}, ..., \mu_{i2n}, ..., \mu_{in1}, \mu_{in2}, ..., \mu_{inn}) \]

\[ \sum_{j=1}^{n} \mu_{ij} = 1, \hspace{1cm} j = 1, 2, ..., n \]  \hspace{1cm} (15)

(ii) Let the objective function of FCM be the fitness function as follows,

\[ J_{FCM}(U, A, X) = \sum_{i=1}^{c} \sum_{j=1}^{n} \mu_{ij}^m d_{ij}^2 \]

\[ = \sum_{i=1}^{c} \sum_{j=1}^{n} \mu_{ij}^m ||x_j - a||^2 \]  \hspace{1cm} (16)

where

\[ a = \left( \sum_{j=1}^{n} \mu_{ij}^m \right)^{-1} \left( \sum_{j=1}^{n} \mu_{ij}^m x_j \right), i = 1, 2, ..., c \]  \hspace{1cm} (17)
(iii) Let the first particle, \( \mu_1 \), be the initial value, by using Picard iteration in FCM algorithm we can obtain the local minimum solution, called \( p_1 \).

We can randomly choose the second particle, \( \mu_2 \), as follows
\[
\mu_2 = \text{normalized}\left[p_1 + 2.5r(\mu_1 - p_1)\right]
\]

satisfying
\[
\sum_{i=1}^{c} \mu_{i,j} = 1, \quad 0 \leq \mu_{i,j} \leq 1, \quad i=1,2,\ldots,c, \quad j=1,2,\ldots,n
\]  

(18)

Let the second particle, \( \mu_2 \), be the initial value, by using the FCM algorithm we can obtain the local minimum solution, called \( p_2 \). Let the better of \( p_1 \) and \( p_2 \) be called \( p_g \).

We can randomly choose the second particle, \( \mu_3 \), as follows
\[
\mu_3 = \text{normalized}\left[\mu_3^t\right]
\]

\[
\mu_3^t = p_2 + 2r_1(\mu_2 - p_2) + 2r_2(\mu_2 - p_g)
\]  

(19)

(18) is satisfied
\[
\sum_{i=1}^{c} \mu_{i,j} = 1, \quad 0 \leq \mu_{i,j} \leq 1, \quad i=1,2,\ldots,c, \quad j=1,2,\ldots,n
\]  

(20)

Where \( r_1 \) and \( r_2 \) are generated randomly in the interval \([0,1]\); Similarly we can randomly choose the particle, \( \mu_n \), as follows
\[
\mu_n = \text{normalized}\left[\mu_n^t\right]
\]

\[
\mu_n^t = p_2 + 2r_1(\mu_2 - p_2) + 2r_2(\mu_2 - p_g)
\]  

(21)

satisfying
\[
\sum_{i=1}^{c} \mu_{i,j} = 1, \quad 0 \leq \mu_{i,j} \leq 1, \quad i=1,2,\ldots,c, \quad j=1,2,\ldots,n
\]  

(22)

(iv) The best previous position (which possesses the best fitness value) of particle k is denoted by \( p_k = (p_{k1}, p_{k2}, \ldots, p_{kd}) \), which is also called \( p_{best} \). The index of the best \( p_{best} \) among all the particles is denoted by the symbol \( g \). The location \( p_g = (p_{g1}, p_{g2}, \ldots, p_{gd}) \) is also called \( g_{best} \).

The velocity for the particle k is
\[
v_k(t+1) = w_k(t) + c_1(t)(p_{best}(t) - p_k(t)) + c_2(t)(g_{best}(t) - p_k(t))
\]

(23)

where
\[
w_k(t+1) = \frac{H_k(t+1)}{\sum_{i=1}^{m} H_i(t+1)}
\]

(24)

Where \( w_k \) is the inertia coefficient which is a constant in the interval \([0,1]\), and can be adjusted in the direction of linear decrease, (In this paper \( w = 0.75 \)); \( c_1 \) and \( c_2 \) are learning rates which are nonnegative constants (In this paper, \( c_1 = c_2 = 2 \)); \( r_1 \) and \( r_2 \) are generated randomly in the interval \([0,1]\);

(v) The termination criterion for iterations is determined according to whether the maximum generation or a designated value of the fitness is reached. In this paper, the given converging error is \( \varepsilon = 0.001 \).
where \( g(t+1) = \sum_{i=1}^{n} \left( \sum_{j=1}^{n} \mu_{ij}(t) \right) \), \( i = 1, 2, ..., c \)  \( (25) \)

3 Experiment of Image Data

A real data set of Image with 262144 sample points from the original Image was selected[9]. All of the sample points were assigned to 3 clusters for clustering analysis. The results were shown in Table1 and Figure 1-4.

<table>
<thead>
<tr>
<th>Cluster</th>
<th>Samples size</th>
<th>Colors</th>
<th>Average distance of center</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>188118</td>
<td>Red</td>
<td>83.3725</td>
</tr>
<tr>
<td>2</td>
<td>3382</td>
<td>Green</td>
<td>104.7881</td>
</tr>
<tr>
<td>3</td>
<td>70644</td>
<td>Blue</td>
<td>126.5261</td>
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</table>

The classification accuracies of testing samples were shown in Table 2.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Accuracies (%)</th>
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<tbody>
<tr>
<td>FCM</td>
<td>30.65</td>
</tr>
<tr>
<td>PSO-FCM</td>
<td>54.67</td>
</tr>
<tr>
<td>PPSO-FCM</td>
<td>75.73</td>
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</table>

From the data of Table 2, we found that the FPSO-FCM could obtain the best result, up to 75.73%.

As shown in Fig. 1, We use the Algorithm of FCM, we could obtain the result, 30.65%.

As shown in Fig. 2, We use the Algorithm of PSO-FCM, we could obtain the result, 54.67%.

As shown in Fig. 3, We use the Algorithm of PPSO-FCM, we could obtain the result, 75.73%.

As shown in Fig. 4, The original river-image

As shown in Fig. 5, The river-image classified by 3 colors
4 Conclusions
An improved new fuzzy clustering algorithm is developed to obtain better quality of fuzzy clustering result. The objective function includes the regulating terms about the covariance matrices. The update equations for the memberships and the cluster centers and the covariance matrices are directly derived from the Lagrange’s method. The fuzzy c-mean algorithm is different from the GK and GG algorithms. The singular problem and detecting the local extreme value problem are improved by the Eigenvalue method and the algorithm of Particle Swarm Optimization. Finally, two numerical examples showed that the new fuzzy clustering algorithm (PSO-FCM-M) gave more accurate clustering results than that of FCM algorithm.

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