Optimal Path Planning for DRSSI based Localization of an RF Source by Multiple UAVs

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Abstract—The Radio Frequency (RF) source localization accuracy depends not only on the measurement performance of sensors, but also on the relative location of the sensors and the source. This paper investigates the impact of UAVs, equipped with RSSI sensors, formation and trajectory on the aerial RF source localization performance for Differential Received Signal Strength Indication (DRSSI) based approach in None Line Of Sight (NLOS) propagation condition. To eliminate the need for knowing the power of the signal source the DRSSI approach is applied. The collected measurements in each waypoint are used to estimate the location of the source iteratively by the use of Extended Kalman Filter (EKF). The Cramer-Rao Lower Bound (CRLB), which expresses a lower bound on the variance of any unbiased estimator, is used as the objective function for the proposed algorithm. Due to the complexity of Jacobin calculation to perform global CRLB optimization, the local values of CRLBs in the current waypoint and the next probable waypoints are used to determine the best path. Maximizing the determinant of the inverse of CRLB, i.e. the Fisher Information Matrix, in each measurement instance over the next probable waypoint, minimizes the estimation uncertainty area. The effectiveness of the proposed algorithm is illustrated by Monte-Carlo simulations and compared with the basic bio-inspired approach of going toward the estimated direction of the source.

Keywords—RF source; Differential Received Signal Strength Indicator (DRSSI); Crammer-Rao Lower Bound (CRLB); Extended Kalman Filter (EKF)

I. INTRODUCTION

Aerial localization of radio frequency emitters, which are also called RF sources or targets throughout this paper, has lots of applications especially in search and rescue missions. The speed and wide vision of the aerial vehicles, such as UAVs or aircrafts, enables effective and fast localization. Flying above the ground level reduces the uncertainty of signal propagation caused by obstacles and improves detection of the signal properties. Different localization approaches use different parameters of the propagated signal and in all of the methods, reducing the interferences and noises improve the localization accuracy.

Localization of an RF source could be classified based on the measurement parameters. Received Signal Strength (RSS) [1], Differential Signal Strength (DRSS) [2], Angle Of Arrival (AOA) [3], Time Of Arrival (TOA) [4], and Time Difference Of Arrival (TDOA) [5] are different measurements of the received signal which have been used to localize the RF source. And still there are efforts to improve the accuracy in different scenarios. RSSI based localization is preferred in many applications including localization in sensor networks [6-7], RF signal localization [8-9] and location based services in cellular phone networks [10] because of the simplicity in hardware and implementation. Given any localization approach, it is well accepted that the arrangement of sensors or sensing points, i.e. waypoints, affects the performance of localization [11-15].

To improve the localization accuracy, in addition to choosing the appropriate method, the proper sensors, and the placement of the sensors in locations with a direct vision of the signal (LOS), the relative position of the sensors and the emitter plays an important role. In the aerial localization with multiple UAVs and a single stationary emitter, which is considered in this paper, the waypoints of UAVs are selected in such a way that the relative geometry of the estimated location of the target and the waypoints to be optimal. The criterion which is used in several papers for this optimization is Cramer-Rao lower bound (CRLB) of the estimation [16-19]. The CRLB is the inverse of Fisher Information Matrix and shows the uncertainty region of the localization.

In [16], an optimal path planning for multiple UAVs with a
heterogeneous mix of sensors is presented. The path planning approach, assign a group of the UAVs for localizing each detected target and adjust their steering to maximize the determinant of Fisher information Matrix. Initially, gradient based steering algorithm is used for locally maximization of the Fisher Information Matrix (FIM); however, in complementary research [17], the nonlinear programming solution for maximizing the determinant of FIM is used to overcome the inaccuracies associated with the first approach. In [1], a multi-UAV path planning approach for RSSI based localization in NLOS condition is proposed to generalize the number of UAVs and develop the NLOS model of propagation.

A similar issue has been considered in wireless sensor network applications. In this type of researches the relative receiver-transmitter geometry and the effect of this geometry on the potential localization performance is investigated. A deep analysis of the optimal sensor-target geometries for range-only, time-of-arrival-based and bearing-only localization has been done in [18]. In [19] the optimality of the relative sensor-emitter geometry for signal strength based localization has been explored.

To the best of the authors’ knowledge, there is no research addressing the optimal path planning in NLOS condition for Differential RSSI (DRSSI) based localization. Due to the independency of DRSSI measurements to the transmitted power, aerial DRSSI based localization has several applications such as localization of time-varying unknown transmitted power which needs multiple UAVs.

In this paper, multiple UAVs are equipped with RSS sensors to localize an RF source with unknown transmitted power in the NLOS propagation condition. In each time intervals, the UAVs form DRSSI measurements and improve the estimated location of the source by using Extended Kalman Filter (EKF). Using the last estimated location of the target and based on the Cramer-Rao lower bound (CRLB) criterion, a local path planning approach guides the UAVs to an optimal configuration in the next time step to reduce localization uncertainty. The next waypoints for the UAVs are determined by maximizing the determinant of FIM, i.e. the inverse of CRLB matrix. This approach is compared to the basic bio-inspired approach of going toward the estimated direction of the source to evaluate the effectiveness of the proposed algorithm.

II. REVIEW ON DRSSI APPROACH

If the RF source power is assumed constant and unknown, two methods can be applied for localization. In the first approach in addition to the location of the target, the transmitted power should also be estimated using the RSSI based localization approach. In the other approach, the difference between two RSS measurements are used to remove the need for transmitted power estimation due to the independency of the DRSSI approach of the transmitted power [20]. It seems that the accuracy of DRSSI approach excels the RSSI based one. On the other hand, localization of sources with unknown time-varying transmitted power can be done by the DRSSI based method only. Therefore, the DRSSI based approach is preferred in many applications.

DRSSI localization utilizes the difference between the received signal strength by multiple UAVs. In each waypoint, UAVs sense the RSS and form the DRSS measurements. In two-dimensional coordinates Each DRSS measurement defines a circle as the locus of possible locations of emitter [21], and the emitter location is specified by the intersection of multiple DRSS circles. For two-dimensional localization at least two DRSS observations are necessary to estimate the source location specifically.

In noiseless environment, the Received signal strength is the transmitted power minus the path loss all in dB as (1). The path loss is a function of the distance between the transmitter and the receiver. In this paper, the general path loss model is considered as (2).

\[ Pr_{ab} = Pr_{tb} - PL_{ab} \]  
\[ PL_{ab} = PL_{d_0} + 10 \log \left( \frac{d}{d_0} \right) \]  

in which \( d \) is the distance between the transmitter and the source. \( PL_{d_0} \) is the path loss in distance of \( d_0 \). According to the path loss model, for determining the distance between the receiver and the transmitter by using RSS measurements, the transmitted power (\( Pr \)) should be known.

The RSS observations are perturbed by zero mean white Gaussian noise (\( \eta \)) with a standard deviation of \( \sigma \). The relation between observed RSS (\( \tilde{Pr} \)) and the true RSS (\( Pr \)) is given by (3).

\[ \tilde{Pr} = Pr + \eta \]  

To eliminate the need to know the power of the transmitter, DRSSI approach is applied. Subtracting the RSS of \( j^{th} \) UAV at the \( k^{th} \) time step from the RSS of \( f^{th} \) UAV at the same time step provides a DRSSI observation at the \( k^{th} \) instance as (4). Since \( N \) UAVs are considered, \( N-1 \) independent DRSS measurements are available at each time step. In the case that the source is a constant power transmitter, UAVs at different instances can form DRSS observations, but in the varying power case the observations would be limited to each instance. Considering DRSS observations in each time step, results in (4).

\[ dPr_{kj} = Pr_{f} - Pr_{j} \]  

Based on (4) and (1), it is specified that the difference of the RSSs of each two UAVs is equivalent to the difference of path loss in that time step. Writing (4) based on the distance between the transmitter and the UAVs forms (5); in which \( d_i \) is the distance between \( i^{th} \) UAV and the transmitter.

\[ dPr_{kj} = -10 \log \left( \frac{d_j}{d_i} \right) \]  

Since the RSS observations have a zero mean Gaussian distribution with variance of \( \sigma^2 \), the DRSSI observations are
disturbed by zero mean Gaussian noise with variance of $2\sigma^2$. The disturbances of two DRSS measurements, i.e. $v_{ij}^k$ in (5), become correlated if the DRSSs share a same sensor, resulting in a non-diagonal covariance matrix for the noise of DRSS measurements.

$$dP_t_{ij}^k = dP_{t_{ij}}^k + v_{ij}^k$$  \hspace{1cm} (6)

III. NOTATION

The UAVs are considered coplanar with the source; thus, a two dimensional model could be applied. The location of UAVs are known in each measurement instance and is given by $\mathbf{U}_i^k = [x_i^k \ y_i^k]^T$, where $\mathbf{U}_i^k$ refers to the position of $i^{th}$ UAV, $i=1, \ldots, N$, and $k$ is a time index which refers to the instances when the UAVs attempt to observe the RSS data. The true location of the target is represented by $\mathbf{P} = [x_p \ y_p]^T$ which is estimated in each measurement instances. $\hat{\mathbf{P}}_i^k = [\hat{x}_p^k \ \hat{y}_p^k]^T$ is the estimated location of the target in $k^{th}$ instance. The range between the estimated location of the source and the $i^{th}$ UAV in $k^{th}$ time step is shown by $d_i^k = \|\hat{\mathbf{P}}_i^k - \mathbf{U}_i^k\|$. To exclude the pairing problem in DRSS data formation, one of the UAVs would be selected as the reference receiver; i.e., all DRSS measurements are taken with respect to the receiver $\mathbf{U}_1^k$.

IV. KALMAN FILTER DESIGN

As mentioned, the DRSS measurements in dB have Gaussian distribution which made Gaussian filtering a suitable case for DRSSI based localization. Also, due to the nonlinearity of the observation function, Extended Kalman Filter (EKF) is used for emitter localization.

The locations of the UAVs are considered known, and the target location needs to be estimated. The differential RSS measurements are independent of transmitted power, so the state vector in each time step $s^k$ only includes the target location.

$$s^k = [x_p^k \ y_p^k]^T$$ \hspace{1cm} (7)

Due to the stationary assumption of the target, the motion model, i.e. the relation between two consecutive states, is as (8):

$$s^k = \begin{bmatrix} x_p^k \\ y_p^k \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} s_p^{k-1}$$ \hspace{1cm} (8)

In this filter, DRSS measurements between two UAVs in each time step are used. To clarify the relation between DRSS measurements as observation equation and the location of RF source, (5) is rewritten in (9).

$$z_{ij}^k = -d_{ij}^k P_{ij}^k$$

$$= -10\lambda \log \frac{\sqrt{(x_p^k - x_j^k)^2 + (y_p^k - y_j^k)^2}}{\sqrt{(x_p^k - x_i^k)^2 + (y_p^k - y_i^k)^2}}$$ \hspace{1cm} (9)

Applying the EKF needs linearized model based on Jacobin (10).

$$H_{i,j}^k = \frac{\partial z_{ij}^k}{\partial s_{i=j}^k} = [H_{i,j,1}^k \ H_{i,j,2}^k]$$ \hspace{1cm} (10)

where:

$$H_{i,j,1}^k = \frac{\partial z_{ij}^k}{\partial x} \bigg|_{x=x^k} = -10\lambda \times \frac{a(b^2 + d^2) - b(a^2 + c^2)}{(b^2 + d^2)(a^2 + c^2)}$$ \hspace{1cm} (11a)

$$H_{i,j,2}^k = \frac{\partial z_{ij}^k}{\partial y} \bigg|_{y=y^k} = -10\lambda \times \frac{c(b^2 + d^2) - d(a^2 + c^2)}{(b^2 + d^2)(a^2 + c^2)}$$ \hspace{1cm} (11b)

in which $a$, $b$, $c$, and $d$ are defined as (12);

$$a = \hat{x}_p^k - x_j^k$$

$$b = \hat{x}_p^k - x_i^k$$

$$c = \hat{y}_p^k - y_j^k$$

$$d = \hat{y}_p^k - y_i^k$$ \hspace{1cm} (12)

Considering an equivalent measurement noise variance at each RSS sensors, the DRSS covariance matrix would be as follows.

$$R_d = \sigma^2 \begin{bmatrix} 2 & 1 & \cdots & 1 \\ 1 & 2 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 2 \end{bmatrix}_{N-1 \times N-1}$$ \hspace{1cm} (13)

V. PATH PLANNING APPROACH

The aim of path planning in an RF source localization problem using multiple UAVs is attaining to the lowest estimation error. As mentioned, the lowest estimation error is described by the lowest variance of estimation errors, which is limited by Cramer-Rao Lower Bound (CRLB). Due to the complexity of Jacobin calculation in the optimization of the CRLB, the local values of CRLBs are used. Therefore, the paths of UAVs should minimize the CRLB of the estimation in each instant, i.e. in each waypoint.

1. Problem description

Assume there are a number of RF sources in an area and multiple networked UAVs attempt to localize each one after another, thus the UAVs in each moment, concentrate on one source and after localization of the source start to localize another one. The sources transmit RF signal which could be
sensed by each of the UAVs. Because of the proximity of the UAVs to the sources and to each other, time delays are negligible, and in each instance, the UAVs could form the DRSS measurements.

The UAV path planning problem involves the determination of UAV waypoint at discrete time instants. The proposed UAV waypoint update algorithm is as follows.

\[ U_i^{k+1} = U_i^k + S_i^k \]  

(14)

where the control vector \( S_i^k \), that shows the displacement of UAVs, satisfies the speed and steering constraint of the UAVs mentioned in (15). \( T \) is the time interval between each consecutive RSS sampling.

\[
[S_i^k] = vT \\
[\zeta U_i^{k+1} - \zeta U_i^k] \leq \varphi
\]  

(15)

\( \varphi \) shows the maximum turn rate at speed \( v \) and without loss of generality the speed of the UAVs considered equivalent to each other and constant in all the instances. In addition to turn rate and speed constraints, some hard constraints which prevent collision between UAVs should be defined. Besides, due to the direction of the transmission and the altitude of the UAVs, in order to receive the signal over than a given threshold, a certain minimum distance between the UAVs and the transmitter should be maintained.

\[
r_{\min} \leq ||U_i^k - U_j^k|| \leq r_{\max}, i,j \in \{1,2,\ldots,N\}, \ i \neq j \\
d_{\min} \leq ||P - U_i^k|| \leq d_{\max}, \ i \in \{1,2,\ldots,N\}
\]  

(16)

Fig. 2, shows the UAVs’ path and constraint on turn rate and the distances. The circles show the waypoints and the dotted line illustrates the UAVs’ path.

2. Cramer-Rao Lower Bound

The Cramer-Rao Lower Bound (CRLB) states that, the inverse of the Fisher information Matrix (FIM) is the lower bound of the variance of any unbiased estimator. Since the probability density function has Gaussian distribution, the Fisher information would be as (17).

\[ I(\hat{\theta}) = J^T R_d^{-1} J \]  

(17)

where \( J \) is the Jacobian matrix and \( R_d \) is the covariance matrix of DRSS measurement error (13). In each time step, FIM would be formed. The Jacobian in \( k^{th} \) time step would be as (18).

\[
J = \frac{10\lambda}{\ln(10)} \begin{bmatrix}
\frac{\hat{x}_p^k - x_i^{k+1}}{(d_i^{k+1})^2} & \frac{\hat{x}_p^k - x_i^{k+1}}{(d_i^{k+1})^2} & \frac{\hat{y}_p^k - y_i^{k+1}}{(d_i^{k+1})^2} & \frac{\hat{y}_p^k - y_i^{k+1}}{(d_i^{k+1})^2} \\
\vdots & \vdots & \vdots & \vdots \\
\frac{\hat{x}_p^k - x_i^{k+1}}{(d_i^{k+1})^2} & \frac{\hat{x}_p^k - x_i^{k+1}}{(d_i^{k+1})^2} & \frac{\hat{y}_p^k - y_i^{k+1}}{(d_i^{k+1})^2} & \frac{\hat{y}_p^k - y_i^{k+1}}{(d_i^{k+1})^2}
\end{bmatrix}
\]  

(18)

The determinant of FIM is used as the objective function for path planning. To determine the UAVs next waypoints, the FIM would be maximized over the waypoints in the next time step. In (18) the superscript \( k+1 \) represents the next time step and \( d_i^{k+1} \) represents the range between the \( i^{th} \) UAV in next waypoint and the last estimation of the source location.

There is a set of possible directions for the UAVs, which produce the choice of next waypoints of the group. The choice causes the determinant of the FIM and consequently the estimation accuracy to be maximized.

Considering desired constraints in (15), in the problem, \( S_i^k \) in (14) would be defined in terms of angular directions \( \theta_i^k \) as (19).

\[
S_i^k = vT \begin{bmatrix}
\cos \theta_i^k \\
\sin \theta_i^k
\end{bmatrix}, \ i = 1,2,3
\]  

(19)

The cost function \( I(\hat{\theta}) \) measures the uncertainty of the localization as a function of the UAVs’ positions \( U_i^k \), the control vectors \( S_i^k \), and the estimated location of the target, i.e. \( \hat{P}_k \), at time \( k \). Based on this formulation, The path planning approach can be formulated as follows.

\[
\hat{\theta}_i^k = \arg \max_{\theta_i^k} \frac{\cos \theta_i^k}{\sin \theta_i^k} \quad i = 1,2,3
\]  

(20)

\[
S_i^k = vT \begin{bmatrix}
\cos \hat{\theta}_i^k \\
\sin \hat{\theta}_i^k
\end{bmatrix}, \ i = 1,2,\ldots,N
\]  

(21)

where \( \hat{\theta}_i^k \) are the optimal angular direction for steering the UAVs to the next waypoints with a higher determinant of fisher information. The proposed algorithm maximizes \( I(\hat{\theta}) \) over \( \hat{\theta}_i^k \) at each waypoint update. Regardless of the minimization approach, a good initialization should be required to avoid convergence to a local minimum. At the initial time, UAVs start to move toward the target due to the incomplete observations for computation of the FIM.

VI. SIMULATION

The proposed approach has been simulated in NLOS condition with three UAVs. The source is located in a random location in a circular search area with five Km of radius. The initial estimation of the source location in EKF is considered
Fig. 3. Path planning for three UAVs. (a) Proposed CRLB based approach. (b) Basic bio-inspired approach. (c) Reduction of the absolute values of the estimation errors in both approaches.

The initial uncertainty of localization, i.e. the covariance of estimation in EKF, is set in such a way that the entire of the search area is covered. The UAVs search the area with the speed of 150 km/h and attempt to measure the DRSS every five seconds. Estimation of the location of the source is updated based on the DRSS measurements and the proposed method determines the next direction of each UAV. The standard deviation of shadowing, which disturbs the RSS measurements, is considered 7dB. The initial UAV locations at the beginning of the localization mission in supposed virtual coordinate system are:

\[
U_1 = [1 \ 1]^T \text{ km} \\
U_2 = [2 \ 0]^T \text{ km} \\
U_3 = [1 \ -1]^T \text{ km}
\]

Fig. 3, shows the paths of UAVs in RF source localization using the proposed approach and bio-inspired approach and the reduction of the absolute values of the estimation errors. In the proposed approach (Fig. 3(a)) the UAVs initially are not on the path toward the source, rather they move away from each other for better localization. After more than half-way through, the UAVs on the both sides of the central UAV gradually approach the estimated location of the source and eventually reach the target.

Fig. 3(b) shows the basic bio-inspired approach, whereby each UAV is steered directly towards the estimated emitter location. The path through the estimated location of the source is determined in each instance, therefore the overall resulted route is not straight. Fig. 3(c) shows the decreasing of the error for CRLB-based optimal and bio-inspired paths. As it is shown in Fig. 3(b), in bio-inspired approach, it is probable that
the UAVs and the source locate on a straight line which is the worst relative position for localization and may cause divergence. The convergence of the CRLB-based path planner is nearly complete and it does not depend on the initial relative position of the UAVs and the target, in contrary the convergence of the bio-inspired algorithm greatly depends on the initial relative position.

The effectiveness of the proposed method in the mentioned condition is evaluated in the Monte-Carlo condition. The Monte-Carlo simulation is based on the averages of 1000 consecutive iterations. Table 1, shows the RMSE of the last estimated location of the source for both CRLB-based and bio-inspired approaches.

Table 1. The RMSE of the last estimation of the RF source location in 1000 consecutive iterations of the CRLB-based and bio-inspired approaches.

<table>
<thead>
<tr>
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<th>CRLB-based approach</th>
<th>Bio-inspired approach</th>
</tr>
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<tbody>
<tr>
<td>RMSE (km)</td>
<td>0.23</td>
<td>1.12</td>
</tr>
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</table>

VII. CONCLUSION

In this paper, a path planning approach is introduced for RF source localization using multiple UAVs. Minimizing the determinant of an approximated CRLB at each UAV's waypoints determine the next waypoints. Extended Kalman filter was used to localize the source. Comparison of the proposed CRLB-based and basic bio-inspired approaches confirmed the effectiveness of the new proposed approach. Comparing the rate of decreasing the estimation error of these two approaches indicated that the localization by the proposed approach converge the true location with higher rates. Also, the convergence of the proposed algorithm is ensured more than the basic approach. Producing continues motion is another advantage of the proposed path over the basic approach.

One of the future researches is to modify the objective function to support non-equal norm of velocity vectors of UAVs. Minimizing the FIM with variable UAV speed would put the UAVs in locations with lower estimation uncertainty. Furthermore, the algorithm should be developed and analyzed for localization of the RF sources with fixed unknown transmitted power by two UAVs. Using the proposed approach for multiple RF sources is another important issue for the future. Finally, the convergence of the approach should be proved.

REFERENCES


