Modeling and Adaptive Sliding Mode Control of Samen SpaceCam

Arash Kiani
School of Electrical Engineering
Iran University of Science and Technology
Tehran, Iran
arashkiania@elec.iust.ac.ir

Seyed Kamal-eddin Mousavi Mashhadi
School of Electrical Engineering
Iran University of Science and Technology
Tehran, Iran
sk_mousavi@iust.ac.ir

Abstract—This paper introduces a three degree-of-freedom cable-suspended robot driven by four cables which is called Samen SpaceCam. This system is widely being used in stadium complexes. First we derive kinematics and dynamic equations of this robot. To overcome the system uncertainties sliding mode control is presented. In this controller to avoid calculating an upper bound of the system uncertainties, an adaptive sliding gain is proposed. Also, stability of closed-loop of the robot system based on Lyapunov theory is proven. Lastly, the proposed controller properties such as good performance tracking, disturbance rejection, and insensitivity to parameter variations are analyzed by simulation.

I. INTRODUCTION

The most important advantages of cable-driven parallel robots (CDPR) are fast motion, light weight, workspaces, heavy loads capabilities, low design, and construction costs [1]. These robots mainly consist of a base, several motors, and an end-effector connected to the base through flexible cables. By means of these motors, the cable lengths are controlled, allowing the end-effector control. Although the cable-driven parallel robots have a short history, a load of research has been done on their performance. RoboCrane is one the first systems that in them the cable is used as a manipulator. This robot was developed at National Institute of Standards and Technology (NIST) [2]. In 1995, Campbell et al. introduced Charlotte robot. This robot was developed in McDonnell Douglas and also was designed for use in space environments [3]. Alp and Agrawal described kinematic and dynamic models, workspace and trajectory planning, for these robots [4]. In [5] dynamics of cable robot with ideal rigid cables is introduced and robust PID control algorithm is proposed for this model. Khosravi et al. dynamic model is extended for elastic cables, and a control strategy is developed using singular perturbation theory. Several control methods have been proposed for parallel manipulators [6-15]. The simplest control structure that can be used to control the position of a parallel manipulator is probably the decentralized PID-control scheme [16]. However, only a few of the proposed methods can be implemented in cable driven parallel manipulators. Most of the proposed control schemes are based on dynamic model of the robot [17]. In order to take advantage of the available information on the platform’s dynamics, e.g. feedback linearization, also known as inverse-dynamics control (IDC) can be used [18, 19]. As an another example, two control algorithms based on inverse dynamics in the workspace and joint space coordinates are analyzed by Gholami et al. [8]. Also, Zollo et al. have presented the theoretical description of two proposed control schemes, the compliant control systems with self-regulating compliance in Cartesian space and in joint space respectively [9].

In this study, we introduce a 4-cable suspended CDPR prototype called Samen SpaceCam. Next, the dynamic and kinematics equations of the robot are derived. Then, a sliding mode control with an adaptive law based on Lyapunov theory (ASMC) for the cable-suspended robot is presented. Finally, properties of ASMC such as good performance tracking, disturbance rejection, and insensitivity to parameter variations are examined by simulation.

II. SAMEN SPACECAM

The Samen SpaceCam robot is a four wire cable-array system covering a playing area of a stadium by computer-controlled cable-drive system. The camera position can be controlled by three cables; the fourth cable is used to increase the available area of operation. This means that at any one time there is a redundant cable; therefore a strategy is required to deal with cable slackness [20]. The geometry of the Samen SpaceCam robot is depicted in Fig. 1.

![Fig. 1: The geometry of Samen SpaceCam](image)

A. System kinematics

The position of each winch pulley is known a priori and with the cable length estimates provided by appropriately scaling the motor encoder count, the problem of estimating the end-effector position in the work space is one of trilateration [20]. For the case of treating the end-effector as a point mass,
the length of each cable can be described in terms of the position of the camera and the position of the associated pulley point [21]:

\[ L_i^2 = (x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2 \] (1)

where \((x,y,z)\) is the end-effector position and \((x_i,y_i,z_i)\) is the position of the top of the ith pulley.

In the Fig. 1, the origin of the coordinate system is defined with respect to the center of the ground-plane of the workspace [20]:

\[ (x_1,y_1,z_1) = (-\frac{A}{2}, 0, H) \]
\[ (x_2,y_2,z_2) = (\frac{A}{2}, 0, H) \]
\[ (x_3,y_3,z_3) = (-\frac{A}{2}, 0, H) \]
\[ (x_4,y_4,z_4) = (\frac{A}{2}, 0, H) \] (2)

Given the equations (2) and (1), the lengths of cables are calculated based on the following equations:

\[ L_1 = \sqrt{(x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2} \]
\[ L_2 = \sqrt{(x-x_2)^2 + (y-y_2)^2 + (z-z_2)^2} \]
\[ L_3 = \sqrt{(x-x_3)^2 + (y-y_3)^2 + (z-z_3)^2} \]
\[ L_4 = \sqrt{(x-x_4)^2 + (y-y_4)^2 + (z-z_4)^2} \] (3)

The above equation is called inverse kinematics. Equations (3) can be used to pre-calculate the required cable lengths for a given the end-effector position. In effect, the cable lengths will form the state vector for the control system. Also, from equations (3) relation between the cable lengths can be written as:

\[ L_1^2 + L_2^2 = L_3^2 + L_4^2 \] (4)

Solving for the direct kinematics requires a choice of which cables are active, as the system is over-constrained. Here we choose cables 1, 2, and 3 for which \((x,y,z)\) are given by [20]:

\[ x = \frac{L_2}{L_1} (L_1 - L_2) \]
\[ y = \frac{L_3}{L_1} (L_1 - L_3) \]
\[ z = H - \sqrt{L_1^2 - (x+\frac{A}{2})^2 - (y+\frac{A}{2})^2} \] (5)

Note that the equation for \(z\) is left in terms of \(x\) and \(y\) and the solution taken for \(z\) ensures that \(z \leq H\). The relation between \(L_i\) and \(x, y, z\) can be defined as [22]:

\[ J = \begin{bmatrix} \frac{\partial L_i}{\partial x} & \frac{\partial L_i}{\partial y} & \frac{\partial L_i}{\partial z} \end{bmatrix}, \quad i=1,2,3,4 \] (6)

where Jacobian matrix \(J\) is:

\[
\begin{bmatrix}
(\frac{A}{2}) & (\frac{A}{2}) & (z-H) \\
L_1 & L_1 & L_1 \\
L_2 & L_2 & L_2 \\
L_3 & L_3 & L_3 \\
L_4 & L_4 & L_4 \\
\end{bmatrix}
\]

(7)

B. System dynamic

The general dynamic equations of motion can be obtained from the Lagrangian formulation. The generalized coordinates are \(q = (x,y,z)^T\). Lagrange’s equation can be written in the form of potential energy, kinetic energy, and generalized forces or torques in the following form [6]:

\[ \frac{d}{dt} \frac{\partial T(q, \dot{q})}{\partial \dot{q}} - \frac{\partial T(q, \dot{q})}{\partial q} + \frac{\partial V(q)}{\partial q} = \tau \] (8)

These results in a kinetic energy of the end-effector which can be written in Cartesian coordinates as:

\[ V = \frac{1}{2} m v^2 \] (9)

where \(v = (\dot{x}, \dot{y}, \dot{z})\) and \(m\) is the mass of the end-effector. In the present study, each cable is assumed to be a force element. Therefore, the potential energy of the system is owing to gravitational forces. The potential energy takes the form:

\[ V = mg z \] (10)

where \(g\) is the gravitational acceleration. It is noted that there is a relation between externally applied wrench on the end-effector and the cable tensions required to keep the system in equilibrium [4], this relationship is as following:

\[ [F_x, F_y, F_z]^T = -J \tau \] (11)

where the external force on the end-effector at the reference point is \(F_x, F_y, F_z\). \(\tau\) is the vector of cable-driven tensions.

Through a series of transformations, substitutions and simplifying the resulting expression, here, we write the equations of motion in terms of \(q\) generalized coordinates as the following general form [6]:

\[ M(q) \ddot{q} + C(q, \dot{q}) \dot{q} + G(q) + \tau = \tau \] (12)

where \(M(q)\) is the inertia matrix of the system, \(C(q, \dot{q})\) is the vector of Coriolis and centripetal terms, \(G(q)\) is the vector of gravity terms, \(\tau_d\) is the vector of external disturbance terms (e.g., random wind, etc.). Here \(\tau\) is the vector of cable-driven tensions \(\tau_i\) for \(i=1,2,3,4\). For Samen SpaceCam robot we can write as follows:

\[ M(q) = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix}, \quad C(q, \dot{q}) = 0, \quad G(q) = 0 \] (13)

III. ADAPTIVE SLIDING MODE CONTROL (ASMC)

A simple approach to robust control is the so-called sliding control methodology. Intuitively, it is based on the remark that it is much easier to control 1st-order systems, be they nonlinear or uncertain, than it is to control general \(n\)-th-order systems. Accordingly, a notational simplification is introduced, which, in effect, allows \(n\)-th-order problems to be replaced by equivalent 1st-order problems [23].

Consider the dynamic equations of Samen SpaceCam:

\[ \ddot{q} = M^{-1}(G(q) - J \tau) \]

\[ = -M^{-1}G(q) + u \]

where \(F = (F_x, F_y, F_z)^T = -M^{-1}J \tau\) and

\[ f(x,y,z)^T = -M^{-1}G(q) \]. Indeed, interaction between \(x, y\) and \(z\) positions is eliminated by choosing \(F = M^{-1}(J \tau)\).

Therefore we can write for \(x\) position:
\[ \ddot{x} = f_x + F_x \] (15)

Also, we can write for \( y \) and \( z \) positions:
\[ \ddot{y} = f_y + F_y \]
\[ \ddot{z} = f_z + F_z \] (16)

We define sliding surface for \( x \) position as follows:
\[ S_x(t) = \ddot{x} + \lambda_x \ddot{x}, \quad \ddot{x} = x - x_d \] (17)

where \( \lambda_x \) is a positive constant. Differentiate respect to time from the sliding surface and using equation (15), we then have:
\[ \dot{S}_x = \dddot{x} + \lambda_x \ddot{x} = f_x + F_x - \ddot{x} + \lambda_x \ddot{x} \]

thus,
\[ \ddot{F}_x = \left[ f_x - \ddot{x} + \lambda_x \ddot{x} \right] \] (19)

where \( f_x - \ddot{x} + \lambda_x \ddot{x} \leq f_A \) and \( f_x \) estimated as \( \dot{F}_x \). Control signal can be obtained for \( x \) position:
\[ F_x = \ddot{F}_x - \beta(t) \text{sgn}(S_x) \text{ or } F_x = \ddot{F}_x - \beta(t) \text{sat}(S_x / \Phi_x) \] (20)

where \( \beta(t) \leq \Phi_x \) and the switching gain is adapted according to the following updating law:
\[ \beta(t) = \gamma(t) \beta(t-1) \] (21)

where \( \gamma(t) \) is a positive constant that let us choose the adaptation speed for the sliding gain [24]. Now define the Lyapunov function candidate:
\[ V = \frac{1}{2} S_x^2(t) + \frac{1}{2} \beta^2(t) \]

where \( S_x(t) \) is the sliding variable defined previously and \( \beta = \ddot{\beta} - \beta \). Differentiate respect to time from the Lyapunov function candidate:
\[ \dot{V} = S_x \dot{S}_x + \ddot{\beta} \beta \]

we then substitute \( \dot{F}_x \) from (20):
\[ \dot{V} = S_x \left[ f_x + F_x - \ddot{x} + \lambda_x \ddot{x} \right] + \dddot{x} \beta(t) \text{sgn}(S_x) \]

\[ \dot{V} = S_x \left[ f_x + F_x - \ddot{x} + \lambda_x \ddot{x} \right] + \dddot{x} \beta(t) \left( \text{sgn}(S_x) - \lambda_x \ddot{x} \right) \]

\[ \dot{V} = S_x \left[ f_x - \ddot{x} + \lambda_x \ddot{x} \right] + \dddot{x} \beta \]

where \( \eta = \text{a strictly positive constant} \). The above equation is obtained by assuming \( \beta_x > f_A + \eta \). Then \( \dot{V} \leq 0 \).

It is worth mentioning that, in the proof have been used the equations (19), (20) and (21). The derivative of \( V \) with respect to time is negative semi-definite but not negative definite. Since \( \dot{V} > 0 \) and \( \dot{V} \leq 0 \), we have \( V(t) \leq V(0) \leq \infty \). Thus \( V(t) \) is bounded, and so its arguments \( S_x(t) \) and \( \ddot{\beta}(t) \) must be bounded also. According to equations (18) and (19), \( \dot{S}_x \) is bounded. Also, according to equation (21) we can say \( \dot{\beta}(t) \) is bounded. Then, differentiate respect to time from (23):
\[ \ddot{V} = S_x \dddot{x} + \dddot{\beta} \beta + \dddot{\beta} \beta \] (25)

On the other hand, if we differentiate respect to time from equations (18) and (21), we conclude that \( \dot{S}_x \) and \( \dddot{\beta} \) are bounded. Therefore \( \dot{V} \) is bounded; hence \( V \) is uniformly continuous.

Under these conditions, since \( \dot{V} \) is bounded, \( \dot{V} \) is uniformly continuous function, so Barbalat’s lemma let us conclude that \( \dot{V} \to 0 \) as \( t \to \infty \), which implies that \( S_x(t) \to 0 \) as \( t \to \infty \).

Therefore \( S_x(t) \) tends to zero as the time \( t \) tends to infinity. Moreover, all trajectories starting off the sliding surface \( S_x(t) = 0 \) must reach it in finite time and then will remain on this surface [23]. The dynamic behavior of the tracking problem (15) is equivalently governed by the following equation:
\[ S_x(t) = 0 \implies \ddot{x} + \lambda_x \ddot{x} = 0 \] (26)

we know that \( \lambda_x \) is a positive constant. Then, the tracking error converges to zero exponentially.

Similarly, the above equation can be obtained for \( y \) and \( z \) positions. Finally, control signals (cable tension forces) for position of Samen SpaceCam end-effector is determined as follows:
\[ \tau = (-J^T) M(q) F \]

where \( F = [F_x, F_y, F_z]^T \), \( M(q) \) is the inertia matrix of the system and \( (-J^T)^T \) is pseudo inverse of \( (-J^T) \). Also, we can attain \( F_x, F_y \) and \( F_z \) from (20).

IV. SIMULATIONS RESULTS

In this section we will study the tracking performance and disturbance rejection of the proposed adaptive sliding mode control by means of simulation examples. It is assumed that, there is an uncertainty around 10 % in the system parameters that will be overcome by the proposed adaptive sliding mode control.

For simulation parameters of Samen SpaceCam are showed in Table 1. Also, we consider the reference trajectory is a circle path [6] with radius of 2 meter as following:
\[ x_f = 2 \cos(0.0167t) \]
\[ y_d = 2 \sin(0.0167t) - 3 \]
\[ z_d = 3 \]

Table 1: Samen SpaceCam parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>m(kg)</th>
<th>A(m)</th>
<th>B(m)</th>
<th>H(m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>25</td>
<td>161.522</td>
<td>149.124</td>
<td>43.731</td>
</tr>
</tbody>
</table>
instead of sgn function. In fact, chattering must be eliminated for the controller to perform properly. This can be achieved by smoothing out the control discontinuity in a thin boundary layer neighboring the switching surface \([23]\):

\[ B(t) = \{ x, \dot{x}, (x, \dot{x}) \leq \Phi \}, \Phi > 0 \]  

(29)

1. Tracking performance of proposed controller:
In this part, tracking performance of Samen SpaceCam via adaptive sliding mode control (ASMC) is studied. Fig. 2, Fig. 3, and Fig. 4 display x, y and, z positions of Samen SpaceCam robot, respectively. As you can see, tracking performance of the proposed controller is very good. Also, Fig. 5 shows tracking desired trajectory error.

Fig. 6 shows velocity of end-effector along the axis x. Besides, Fig. 7 and Fig. 8 show y and z velocities of the end-effector of Samen SpaceCam, respectively. The velocity of end-effector must be smooth that this controller has the ability to smooth the velocity. Cables tension forces are plotted in Fig. 9. The tension forces must be positive. As you can see, these are positive. Also, the values of control signals (tension force of cables) are not big, because the weight of end-effector is approximately 250 N.

2. Disturbance rejection:
For this purpose, we assume that disturbance model is a step function with amplitude 15 N (approximately 10% of maximum tension forces of cables). Indeed, we add a disturbance to tension forces of cables in \( t = 30 \). Tracking desired trajectory error and tension forces of cables in presence of step disturbance with amplitude 15 N are plotted in Fig. 11 and Fig. 12, respectively. Fig. 11 shows the tracking error for \( t = 20 \) to 100. Also, the tracking error for \( t = 0 \) to 100 is plotted in Fig. 10. According to Fig. 11 step disturbance is rejected well by the proposed controller. Maximum tracking error in the presence of this disturbance is 5 cm. Fig. 12
presents tension forces via SMC in presence of this disturbance.

![Fig. 10: Tracking error in presence of step disturbance via ASMC](image1)

For $t = 0$ to $100$

![Fig. 11: Tracking error in presence of step disturbance via ASMC](image2)

For $t = 20$ to $100$

![Fig. 12: Tension forces of cables in presence of step disturbance via ASMC](image3)

3. Parameters variation:

As previously mentioned, it is assumed that there is an uncertainty around 10% in the system parameters. Fig. 13 demonstrates the tracking error with uncertainty in the mass of end-effector. This figure shows tracking error convergence to zero. Indeed, the proposed controllers are insensitive to parameter variations. Moreover, tension forces of cables are plotted in the Fig. 14. These tension forces are bigger than tension forces for without uncertainty in the system.

![Fig. 13: Tracking error with uncertainty in the mass end-effector via ASMC](image4)

4. Comparing ASMC with sliding mode control (SMC)

In this section, we compare ASMC and sliding mode control (SMC) together. Fig. 15 demonstrates the position of Samen SpaceCam via the proposed controller (ASMC) and SMC. It can be seen that the tracking performance both of them are good. The velocity of end-effector of Samen SpaceCam and tension forces of cables via ASMC and SMC are plotted in Fig. 16 and Fig. 17, respectively. As you can see, the velocity of end-effector via ASCM is smaller than SMC. ASMC has better performance than SCM, because the velocity of end-effector is very important for Samen SpaceCam.

![Fig. 15: The position of Samen SpaceCam via ASMC and SMC](image5)

![Fig. 16: Velocity of end-effector via ASMC and SMC](image6)

The control signal (tension forces of cables) of our proposed variable structure control schemes is smaller than the control signal of the traditional variable structure control schemes. Because in the last one the sliding gain value should be chosen high enough to overcome all the possible uncertainties that could appear in the system along the time [24].
V. CONCLUSION

This paper introduces a three degree-of-freedom cable-suspended robot driven by four cables which is called Samen SpaceCam. We present an adaptive sliding mode control to overcome the uncertainties of system and to avoid estimating an upper bound of the system uncertainties. The proposed controller ensures stability of closed-loop system and the tracking error converges to zero exponentially. Afterwards, the proposed controller properties such as good performance tracking, disturbance rejection, and insensitivity to parameter variations are demonstrated by simulation. One of the most interesting advantages of this controller is that it is independent of the robot parameters in a wide range of variations. Finally, ASMC and SMC are compared together. The ASMC has better performance compared to SMC. The control signal of our proposed variable structure control schemes (ASMC) is smaller than that of the traditional one (SMC) due to the fact that in the latter one the sliding gain value should be chosen high enough in order to overcome all the possible uncertainties that could appear in the system along the time.

REFERENCES