Improving the stability of gait planning for quadruped robots

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Abstract—In this paper the stability of gait planning for the quadruped robot in flat surface is considered, and the effects of planning parameters on the stability of the robot have been examined. Continuous trajectory for the body in forward direction has been derived. The study demonstrates that adding periodic longitudinal (PLON) body motion back and forth and periodic lateral (PLAT) body motion left and right improved the stability margin. Finally this approaches were tested on the TMUBot quadruped, and the results show significant improvement in the robot.

Keywords—Kinematics; Gait planning; Trajectory planning; Static stability;

I. INTRODUCTION

Legged locomotion over rough terrain is more fuel efficient than wheeled locomotion. Some surfaces are inaccessible to wheeled robots and they cannot move properly on them. Thus, although legged robots are complex, their capabilities to adapt on different surfaces attracted much attention. Legged robots may be one leg, biped, quadruped, hexapod robots. Study of the Quadruped robots seriously began from the 1980’s [1]. The main goal of this type of robot is the payload over uneven surfaces.

Stable gait generation for quadruped robot is the main issue. A gait is a manner of walking with parameters such as leg sequence, time of swing and support phase. The gaits that typically use for quadruped robot are crawl, trot, pace and bound gait. The gaits that have low speed and use stability margin criteria are called static gait [2]. In recent years, many studies on stable gait generation have been done. A successive gait-transition method proposed in [3] realizes omnidirectional static walking. In [4] a motion planning algorithm of static walking gait has presented. Quadruped dynamics and stable gait planning have been studied in [5] and explicit dynamic model of a quadruped robot is presented.

In this paper, we consider gait planning of a quadruped robot on an even surface in straight direction and implement this planning on the TMUBot quadruped, designed and implemented in Intelligent Control Systems Laboratory at the ECE School of Tarbiat Modares University. A five degree polynomial in Cartesian space for feet and body motion planning has been used. Each gait divide into three phases, first phase for body movement and other phases for swing leg movement. Extra parameters PLON and PLAT in gait generation are used to improve static stability.

This paper is organized as follows: the design structure and the dynamic model of quadruped are presented in section 2 and 3. In section 4 and 5, trajectory and gait planning with experimental results are addressed. The stability analysis and simulated results are described in section 6, and the concluding remarks are given in section 7.

II. STRUCTURE DESIGNING

This quadruped robot is designed to obtain the posture which is similar to little dog except that it is almost thrice greater than it. Figure 1 and 2 illustrate the picture and schematic of the TMUBot respectively. Each leg is composed of three links and has three active DOF (degrees of freedom). The first revolute joints in each leg moves in lateral direction and the two other joints in forward direction. Body of robot has six DOF, and therefore, the robot has totally 18 DOF.

Fig. 1. The TMUBot quadruped.
III. DYNAMIC MODEL

A. Kinematic Model

As seen in Figure 2, two coordinate systems A and B are considered. The coordinate system A is fixed and is connected to ground and the coordinate system B is moving and located in the center of mass of the body. Position vector $P$ is position of coordinate system B respect to coordinate system A, vectors of $X_i (i=1,\ldots,4)$ indicate position of zeros frame $O_i$ connected the coordinate system B, $L_i$ is leg ankle vector(Center of the fourth frame) with respect to the coordinate A, also $P_i$ is ankle vector respect to $O_i$. Accordingly, the following equation applies for the Kinetic of Quadruped:

$$\begin{align} \mathbf{P}^A &= -^A R_B^B X_i -^A R_B^B L_i +^A P_i \end{align}$$

(8)

Where $^A R_B$ and $^{\alpha} R_{\alpha}$ are rotation matrix of the coordinate system B to A and the coordinate system $O_i$ to B.

To describe the motion of each leg, the coordinates are attached as shown in Figure 3. Geometrical dimensions of the robot are shown in Table 1.

![Coordinates located on the body and leg i](image)

**TABLE I
GEOMETRICAL DIMENSIONS OF ROBOT**

<table>
<thead>
<tr>
<th>Symb</th>
<th>Quantity</th>
<th>SI (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>body length</td>
<td>80</td>
</tr>
<tr>
<td>Y</td>
<td>body Width</td>
<td>24.5</td>
</tr>
<tr>
<td>H</td>
<td>body Height</td>
<td>14</td>
</tr>
<tr>
<td>d</td>
<td>Offset of hip</td>
<td>7</td>
</tr>
<tr>
<td>$l_2$</td>
<td>Leg</td>
<td>34</td>
</tr>
<tr>
<td>$l_3$</td>
<td>Shank</td>
<td>30.5</td>
</tr>
</tbody>
</table>

Thus, the transfer matrix of the third coordinate system to the zero coordinate system which is the junction of the leg to the body is obtained by the following equation:

$$^4 T_4 = ^0 T_1^1 T_2^2 T_3^3 T_4 = \begin{bmatrix} c_{c_23} & -c_{s_23} & s_i & ds_{c_2} + c_i (l_2 c_2 + l_3 c_{c_23}) \\ s_{c_23} & c_{s_23} & -c_i & -dc_{s_2} + s_i (l_2 c_2 + l_3 c_{c_23}) \\ s_{23} & c_{23} & 0 & l_2 s_2 + l_3 s_{c_23} \end{bmatrix}$$

Where $c_{c_2}, s_{c_2}, c_{s_2}$ and $s_{s_2}$ are indicated $\cos(\theta_2), \sin(\theta_2), \cos(\theta_2 + \theta_3)$ and $\sin(\theta_2 + \theta_3)$.

B. Inverse Kinematic

Inverse kinematics are used in order to calculate the joint angles. Inverse kinematics for the $i$-th leg can be calculated as follows:

Position vectors $^o P_i (i=1,\ldots,4)$ that connect center of the fourth coordinate system of each leg to the center of the zeros coordinate system.

$$^o P_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}$$

(2)

The third angle is obtained from the following equations:

$$c_{3,i} = \frac{x_i^2 + y_i^2 + z_i^2 - d^2 - l_2^2 - l_3^2}{2l_2 l_3}$$

(3)

$$s_{3,i} = \pm \sqrt{1 - c_{3,i}^2} \Rightarrow \theta_{3,i} = \arctan(s_{3,i}, c_{3,i})$$

(4)

As can be seen, there is a dual solution for the third angle. For the second angle, we find:

$$\theta_{2,i} = \arctan\left(\frac{z_i}{r_i}, \sqrt{\frac{k_{i,j} y_i + y_i^2 - d^2}{r_i^2}}\right) - \arctan(2k_{2,j}, k_{1,i})$$

(5)

where $r_i = \sqrt{k_{i,j}^2 + k_{j,i}^2}, y_i = \arctan(k_{2,j}, k_{1,i})$ and

$$k_{i,j} = l_2 l_{c_{3,i}}, k_{2,j} = l_2 s_{3,j}$$

(6)
And finally the first angle is calculated using the values obtained:

\[ \theta_{ij} = \text{atan2} \left( \frac{dx_i + (c_{i1}l_5 + c_{i2}l_3)y_i}{d^2 + (c_{i2}l_2 + c_{i3}l_4)^2}, \frac{dx_i + (c_{i1}l_5 + c_{i2}l_3)x_i + dy_i}{d^2 + (c_{i2}l_2 + c_{i3}l_4)^2} \right) \]  

(7)

IV. TRAJECTORY PLANNING

For trajectory planning of feet and body motion in forward direction \( x \), a curve with five-degree polynomial is considered in Cartesian space:

\[ C(t) = at^5 + bt^4 + ct^3 + dt^2 + et + f \]  

(9)

The curves for the robot path are designed in three stages:

1. The key poses of the feet and the body of the robot are defined. Key poses are include the center of body and the robot ankles.

2. Parametric path equations are written for each key pose respect to time.

3. Using the inverse kinematic equations the joint angles corresponding to each key pose are calculated.

Trajectory for each leg and the body must be determined in every gait. Assume that only the body of robot moves in the first interval. In this case the legs of the robot are on the ground and only the joint angles and their hip positions changes. Another situation is when one leg is in the swing phase, in which in addition to the previous changes, the corresponding leg ankle moves. Thus, by having the initial and final position, velocity, and acceleration at the initial and final time, one can calculate the coefficients of the 5-degree curve for the leg ankle. When position of the key poses are found, the joint angles are obtained by solving inverse kinematics equation. Figure 4 shows, the experimental test of the described trajectory planner on TMUBot for making trajectory of leg 2 as the swing leg.

Fig. 4. Trajectory leg2 for TMUBot.
Figure 5 shows all joint angles of the leg 2 in three gait sequences. The red, blue and green line indicate joints 3, 1, and 2 respectively.

![Joint angles of leg 2.](image)

Fig. 5. Joint angles of leg 2.

V. GAIT PLANNING

Typical gaits of quadrupeds is shown in figure 6. The number beside leg indicate the beginning time of the swing phase. For example, crawl gait lifts forward left, back right, forward right and back left leg. For a stable gait, the initial posture of the leg is important. As shown in figure 7, we consider leg sequence in the form of: back right, forward right, back left, and forward left, and the initial position of the leg ankles at time zero are 10 cm far from the projection of hip on the ground into the body. Assume that robot moves along x axis. The body can move in x, y, z direction. The parameters considered in the gait path planning in this work include: max step height, max step-length-ratio, body height and max body roll and body pitch. Adjusting these parameters allows the leg ankle be in the feasible area, and the gait generates.

![Sequence and trajectory of the legs.](image)

Fig. 7. Sequence and trajectory of the legs.

As shown in figure 8, each gait divide into three phases, first phase is for the body movements in different directions alone, and other phases for swing leg movements along with the body movements. Leg movement is such that its tip goes up to full step height while moving forward half of the step length. Timing of each phases of a gait is in the form of:

\[
T_1 = 0.3T, \quad T_2 = \frac{(1 - T_1)}{6}, \quad T_3 = \frac{2(1 - T_1)}{6}
\]

(10)

where \(T_1\) is the time duration of body movement while all four legs are on the ground, and \(T_2\) and \(T_3\) are the time duration of swing leg when it goes up and down, respectively and \(T\) is total time interval of a gait. We consider \(T_3\) greater than \(T_2\) so as to reduce effect of impact of the legs, which allows the leg ankle’s speed reduces when it hits the ground.

![Three phases of a gait.](image)

Fig.8. Three phases of a gait.

Also we implemented PLAT motion for body such that when right side legs are in swing phase, body moves in opposite direction and moves left proportional to PLAT value, and similarly when left side legs are in swing phase, body moves right. Moreover, we used PLON motion for body as well such that when back legs are in swing phase, the body moves in forward direction and when the front legs are in swing phase, the body goes back.
VI. STABILITY ANALYSIS

In this section we study the effect of path planning parameters on the stability of quadruped. Stability analysis categorized in two types: static stability and dynamic stability. Static stability margin is used to check static stability. According to the definition, the static stability margin (SM) is the shortest distance from the vertical projection of the center of gravity of the robot to the boundary of the support pattern [7]. Our study shows that adding PLAT and PLAN motion in path planning improve static stability margin. We check this by employing the modified planning on the TMUBot. According this criterion static stability will satisfy if the projection of COG (center of gravity) stays in support polygon in each time. Support polygon is defined as the convex polygon formed by connecting footprints on the ground. In the gait in our study, $\beta$ (Duty factor) is greater than 0.75, and therefore for the times that all four legs are on the ground, the stability margin increases.

We set lateral variations of the body PLAT=2, longitudinal variations of the body PLON=2, and chose the step-length-ratio of 0.2 (meaning that the step-length is 0.2 of the maximum possible step-length) then compared the results with step-length-ratio equal to 1.

Figures 10 and 11 show result for change step-length-ratio from 0.2 to 1 of maximum step-length-ratio with PLON=2 and 0. The blue line is drawn for the step-length-ratio of 0.2 and green line is for the step-length-ratio of 1. As we see from the figures, the stability margin for PLON=2 is improved.

Figures12 show the result for changing PLAT from 2 to 5 with PLON equal to 2 for the robot body movements and Figures13 shows the result for changing PLON from 0 to 2 respectively. As we see with these variations, the stability
margin has been improved. In this figures, the blue line improves stability margin for changes.

As can be seen from the figures Minimum SM for changing step-length-ratio from 0.2 to 1 with PLON of 2 obtained 0.0029(m) and 0.0143(m), respectively. Moreover, minimum SM for changing step-length-ratio from 0.2 to 1 with PLON of 0 are calculated as 0.008(m) and 0.0044(m), respectively. Minimum SM for changing PLAT from 2 to 5 cm with PLON 2 are calculated 0.0168(m) and 0.042(m) respectively and Minimum SM for variation in PLON from 0 to 5 is 0.0156(m) and 0.0168(m) respectively.

VII. CONCLUSION

In this paper, we proposed an improved gait trajectory for TMUBot in terms of stability margin. Generally it is observed that the stability margin can be increased for high step-length-ratio by increasing periodic lateral movements of the robot body using parameter PLON2. It is worth noting that, the leg ankles should be checked to remain in feasible area with these variations.

REFERENCES