# The Nonlinear Suboptimal Diving Control of an Autonomous Underwater Vehicle

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***Abstract*—This work proposes a nonlinear suboptimal diving control scheme for autonomous underwater vehicles (AUVs) in presence of hydrodynamic parameter uncertainties and disturbance. The controller is designed via the state-dependent Riccati equation (SDRE) method that regulates the system's states by minimization of the related quadratic cost function. To compensate the system's restoring force (gravity/buoyancy), the common SDRE control law is supplemented with a discontinuous control law analogous to sliding surface notion. We applied the proposed controller to a well-known AUV model (REMUS). Simulation results demonstrate the effectiveness of the proposed controller under system's parameter uncertainties and disturbance with appropriate regulation and tracking performance.**

***Keywords—SDRE; AUV; Optimal Control; nonlinear Control***

## I. Introduction

Recently, remotely operated vehicles and AUVs have been widely used for scientific, commercial and military purposes. The AUVs have very complex dynamics models which depend on several parameters like added-mass, hydrodynamics damping coefficients and etc. Considering the complicated behavior of AUVs and limits on designing controllers for subsurface vehicles with six DOFs, a number of simplifications have been made such as linearizing and decoupling to reduce complexity. In the scope of this paper, diving control of an AUV is considered for regulation and tracking problems.

The so-called "state-dependent Riccati equation" technique provides a systematic and effective manner for control systems' design for a variety of nonlinear models and systems [1-8]. The SDRE is a well-known strategy and has become very popular over the past decade, providing a very effective algorithm for synthesizing nonlinear feedback control by allowing nonlinearities in the system states while lowering energy consumption in quadratic performance index is regarded.

The SDRE was used for controlling the AUVs, particularly dive plane control. Naik and Singh presented SDRE-based robust dive plane control of an AUV with input constraint [9]. The indirect robust control method was also used for depth control of autonomous underwater vehicles [10]. Yim and Oh utilized the SDRE-based dive plane control of an AUV with control constraint with the assumption that gravity and buoyancy force were negligible and they attenuated it by feedforward compensation in advance [11]. Yan et al. investigated the dive plane trajectory tracking control of an AUV under current disturbance [12]. Jantapremjit and Wilson presented controlling and guidance of homing and docking tasks for an autonomous underwater vehicle [13]. Honget et al. studied depth control of an AUV based on sliding mode control [14]. Cao and Su presented input-output linearization and adaptive control design for dive-plane control of AUVs as well [15, 16].

In this present work, diving control problem of AUVs without ignoring gravity and buoyancy force will be presented. The distribution of gravity in the system will not allow a perfect regulation or tracking by SDRE controller. So, a correction term is added to conventional SDRE for compensating this known disturbance. The proposed modified SDRE regulates the depth to desired value without steady-state error while the conventional SDRE fails.

The rest of the article is organized as follows. A mathematical model of AUV is expressed in Section II. The structure of the controller is presented in Section III. Implementation of the controller on the model is presented in Section IV and the results of simulation are provided in Section V. Finally, conclusions are expressed in Section VI.

## II. Mathematical Model of AUV

A schematic view of an AUV with related coordinate systems is presented in Fig. 1 to show a six-DOF AUV model.

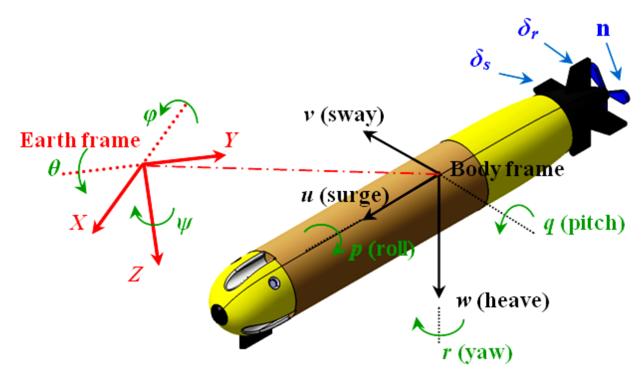


Fig. 1.The schematic of a six-DOF model of REMUS.

The two reference frames are applied to the model: earth-fixed frame and body-fixed frame. Descriptions of the parameters are expressed in Table I. The Attitude to present the kinematics of AUV's model in the global reference frame is based on Euler angles.

TABLE I
THE NOTATION OF SNAME FOR MARINE VESSELS [17]

| Description | Linear and angular velocities | Positions and Euler angles |
|---|---|---|
| Motion in the x-direction(surge) | $u$ | $x_c$ |
| Motion in the y-direction(sway) | $v$ | $y_c$ |
| Motion in the z-direction(heave) | $w$ | $z_c$ |
| Rotation in the x-direction(roll, heel) | $p$ | $\varphi$ |
| Rotation in the x-direction(pitch, trim) | $q$ | $\theta$ |
| Rotation in the x-direction(yaw) | $r$ | $\psi$ |

The kinematics equation is written as:

$$\dot{\boldsymbol{\eta}} = \mathbf{J}(\boldsymbol{\eta})\mathbf{v} = \begin{bmatrix} \mathbf{R}(\boldsymbol{\eta}) & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathbf{T}(\boldsymbol{\eta}) \end{bmatrix} \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{bmatrix}, \quad (1)$$

where $\boldsymbol{\eta} \in \Re^{6\times1}$ is the vector of position and attitude of a vehicle in the inertial frame, $\mathbf{v} \in \Re^{6\times1}$ is the vector of the linear and angular velocities of a vehicle in a fixed-body frame, $\mathbf{v}_1 = [u \quad v \quad w]^T$ and $\mathbf{v}_2 = [p \quad q \quad r]^T$. $\mathbf{R}(\boldsymbol{\eta})$ is the rotation matrix from the body frame to the inertial frame, and $\mathbf{T}(\boldsymbol{\eta})$ is the angular velocity transformation from the body frame to the inertial frame.

The six-DOF dynamics model of the AUV is derived from the Newton-Euler equation of motion of a rigid body in the fluid. The dynamics model is given by [18]:

$$\begin{cases} \dot{\boldsymbol{\eta}} = \mathbf{J}(\boldsymbol{\eta})\mathbf{v} \\ \mathbf{M}\dot{\mathbf{v}} + \mathbf{C}(\mathbf{v})\mathbf{v} + \mathbf{D}(\mathbf{v})\mathbf{v} + \mathbf{G}(\boldsymbol{\eta}) = \boldsymbol{\tau} \end{cases}, \quad (2)$$

where $\mathbf{M} \in \Re^{6\times6}$ is the inertia matrix, $\mathbf{C}(\mathbf{v}) \in \Re^{6\times6}$ is the matrix of Coriolis and centripetal terms, $\mathbf{D}(\mathbf{v}) \in \Re^{6\times6}$ is damping matrix, $\mathbf{G}(\boldsymbol{\eta}) \in \Re^{6\times1}$ is the vector of gravitational/buoyancy force and moments, $\boldsymbol{\tau} \in \Re^{6\times1}$ is the vector of an external force and moments, acting on the AUV.

The general AUV model involves highly nonlinear and coupled equations that lead to a high level of complexity in control design. Most preformed maneuvers by AUVs can be described by three uncoupled basic subsystems: speed, heading and depth, Table II.

The diving subsystem presents the depth motion of the AUV in the X-Z plane. The control input commands the deflection of stern planes or bow planes $\delta_s$. The system state vector in this case is considered as $\mathbf{x}(t) = [\boldsymbol{\eta}(t) \quad \mathbf{v}(t)]^T$ where $\boldsymbol{\eta}(t) = [z \quad \theta]^T$ and $\mathbf{v}(t) = [w \quad q]^T$.

TABLE II
DECOUPLED SUBSYSTEMS OF AN UNDERWATER VEHICLE [18]

| Subsystem | Description | Control Input |
|---|---|---|
| Speed | $u(t)$ | $n(t)$ |
| Steering | $v(t)$, $r(t)$, $\psi(t)$ | $\delta_r(t)$ |
| Diving | $w(t)$, $q(t)$, $\theta(t)$, $z(t)$ | $\delta_s(t)$ |

Assuming a constant speed $u$ for $x_c$ direction and ignoring other states, according to the equation of motion (2), the dynamics model for diving is reduced as:

$$\hat{\mathbf{J}}(\boldsymbol{\eta}) = \begin{bmatrix} \cos\theta & 0 \\ 0 & 1 \end{bmatrix}, \quad (3)$$

$$\hat{\mathbf{M}} = \begin{bmatrix} m - Z_{\dot{w}} & -(mx_G + Z_{\dot{q}}) \\ -(mx_G + M_{\dot{w}}) & I_y - M_{\dot{w}} \end{bmatrix}, \quad (4)$$

$$\hat{\mathbf{C}}(\mathbf{v}) = \begin{bmatrix} -Z_{uw}u & -Z_{uq}u - mu - mqz_G \\ mu - M_{uw}u + mqz_G & -M_{uq}u \end{bmatrix}, \quad (5)$$

$$\hat{\mathbf{D}}(\mathbf{v}) = \begin{bmatrix} -Z_{w|w|}|w| & -Z_{q|q|}|q| \\ -M_{w|w|}|w| & -M_{q|q|}|q| \end{bmatrix}, \quad (6)$$

$$\hat{\mathbf{G}}(\boldsymbol{\eta}) = \begin{bmatrix} \cos(\theta)(B - W) \\ -\sin(\theta)(Bz_B - Wz_G) - \cos(\theta)(Bx_B - Wx_G) \end{bmatrix}, \quad (7)$$

$$\boldsymbol{\tau} = \begin{bmatrix} Z_{uu\delta u}u^2 \\ M_{uu\delta u}u^2 \end{bmatrix} \delta_s, \quad (8)$$

where $W$ is the AUV's weight, $B$ is the buoyancy, $I_y$ is the moment of inertia about y-axis of body frame, $\{Z_{\dot{w}}, Z_{\dot{q}}, M_{\dot{w}}, M_{\dot{q}}\}$ are added mass coefficients, $\{Z_{uw}, Z_{uq}, M_{uw}, M_{uq}\}$ are Coriolis and centripetal coefficients which can be derived by hydrodynamic derivatives, and $\{Z_{w|w|}, M_{q|q|}\}$ are nonlinear damping vortex shedding coefficients that can be estimated by calculating the hull drag [19].

## III. THE SDRE CONTROLLER

The optimal control problem is formulated with the assumption that all state variables are available for feedback. The SDRE approach involves manipulating the dynamics equation:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x}, \mathbf{u}), \quad (9)$$

where the state vector is $\mathbf{x} \in \Re^n$, $\mathbf{u} \in \Re^m$ is the input and $\mathbf{f}(\mathbf{x}) \in \Re^n \to \Re^n$ and $\mathbf{g}(\mathbf{x}, \mathbf{u}) \in \Re^n \to \Re^n$. In the state-dependent coefficient (SDC) form, the following equation will be reached:

$$\dot{\mathbf{x}} = \mathbf{A}(\mathbf{x})\mathbf{x} + \mathbf{B}(\mathbf{x})\mathbf{u}, \tag{10}$$

where $\mathbf{A}(\mathbf{x}):\Re^n \to \Re^{n\times n}$ and $\mathbf{B}(\mathbf{x}):\Re^n \to \Re^{n\times m}$. The aim of this method is to minimize the quadratic index:

$$J = \frac{1}{2}\int_0^\infty \{\mathbf{x}^T\mathbf{Q}\mathbf{x} + \mathbf{u}^T\mathbf{R}\mathbf{u}\}dt. \tag{11}$$

The weighting matrix for states is $\mathbf{Q}:\Re^n \to \Re^{n\times n}$ and for input it is $\mathbf{R}:\Re^n \to \Re^{m\times m}$. $\mathbf{Q}$ must be symmetric, positive semi-definite and $\mathbf{R}$ must be symmetric, positive definite. The SDRE control law is:

$$\mathbf{u} = -\mathbf{R}^{-1}\mathbf{B}(\mathbf{x})^T\mathbf{k}(\mathbf{x})\mathbf{x}, \tag{12}$$

where $\mathbf{k}(\mathbf{x})$ is the positive definite solution of the SDRE as in the following equation:

$$\begin{aligned}&\mathbf{k}(\mathbf{x})\mathbf{A}(\mathbf{x}) + \mathbf{A}(\mathbf{x})^T\mathbf{k}(\mathbf{x}) - \\ &\quad \mathbf{k}(\mathbf{x})\mathbf{B}(\mathbf{x})\mathbf{R}^{-1}\mathbf{B}(\mathbf{x})^T\mathbf{k}(\mathbf{x}) + \mathbf{Q} = \mathbf{0}.\end{aligned} \tag{13}$$

To compensate disturbance, a corrective term is added to control law (12) for eliminating steady-state error. So, the new control law is reformed as

$$\mathbf{u} = \mathbf{u}_{SDRE} + \mathbf{u}_{Corr}, \tag{14}$$

where

$$\mathbf{u}_{SDRE} = -\mathbf{R}^{-1}\mathbf{B}(\mathbf{x})^T\mathbf{k}(\mathbf{x})\mathbf{x} \tag{15}$$

$$\mathbf{u}_{Corr} = -\boldsymbol{\lambda}\,\mathrm{sgn}(\boldsymbol{\sigma}(\mathbf{x})) \tag{16}$$

in which $\mathrm{sgn}(\cdot)$ is a signum function, $\boldsymbol{\lambda}$ is $m\times n$ constant matrix and $\boldsymbol{\sigma}(\mathbf{x})$ is sliding surface. The sliding surface is suggested as:

$$\boldsymbol{\sigma}(\mathbf{x}) = \mathbf{x} - \mathbf{x}_{des}, \tag{17}$$

where $\mathbf{x}_{des}$ is desired state vector.

## IV. Implementation of the Controller on REMUS AUV

An important step in designing controller is state-dependant coefficient parameterization, which changes the system (9) to an SDC form (10). The state vector is considered as $\mathbf{x}(t) = [z \quad \theta \quad w \quad q]^T$. There are some terms in the vector of gravitational/buoyancy force and moments which cannot be directly transformed into SDC form since $\theta$ is argument of the trigonometric functions. So, Taylor series expansion is used for changing the *sine* and *cosine* functions. The factorization is expressed as:

$$\begin{bmatrix}\cos\theta(B-W)\\ -\sin\theta(Bz_B - Wz_G) - \cos\theta(Bx_B - Wx_G)\end{bmatrix} = \begin{bmatrix}0 & g_1\\ 0 & g_2\end{bmatrix}\begin{bmatrix}z\\ \theta\end{bmatrix} + \begin{bmatrix}d_1\\ d_2\end{bmatrix}, \tag{18}$$

where

$$g_1 = (B-W)(-\theta/2 + \theta^3/24), \tag{19}$$

$$\begin{aligned}g_2 = &(Bz_B - Wz_G)(1 - \theta^2/6 + \theta^4/120) - \\ &(Bx_B - Wx_G)(-\theta/2 + \theta^3/24),\end{aligned} \tag{20}$$

$$\mathbf{D}_n = \begin{bmatrix}d_1\\ d_2\end{bmatrix} = \begin{bmatrix}B-W\\ -(Bx_B - Wx_G)\end{bmatrix}. \tag{21}$$

The vector $\mathbf{D}_n$ appears due to Taylor series expansion of $\cos(\cdot)$, and it is assumed as known disturbance. Since there is one actuator or control input in diving model and gravity/buoyancy force and moments are distributed in both two rows of equation of motion, some steady-state errors remain in the responses. The proposed modified structure will help to compensate undesirable distributed disturbance.

The state-space representation of the system is written in the following forms:

$$\mathbf{A}(\mathbf{x}) = \begin{bmatrix}\mathbf{0}_{2\times 2} & \mathbf{a}_{12}\\ \mathbf{a}_{21} & \mathbf{a}_{22}\end{bmatrix}_{4\times 4}, \tag{22}$$

$$\mathbf{B}(\mathbf{x}) = \begin{bmatrix}\mathbf{0}_{2\times 1}\\ \mathbf{b}_{21}\end{bmatrix}_{4\times 1}, \tag{23}$$

where

$$\mathbf{a}_{12} = \hat{\mathbf{J}}(\boldsymbol{\eta}), \tag{24}$$

$$\mathbf{a}_{21} = -\hat{\mathbf{M}}^{-1}\begin{bmatrix}0 & g_1\\ 0 & g_2\end{bmatrix}, \tag{25}$$

$$\mathbf{a}_{22} = -\hat{\mathbf{M}}^{-1}(\hat{\mathbf{C}}(\boldsymbol{\nu}) + \hat{\mathbf{D}}(\boldsymbol{\nu})), \tag{26}$$

$$\mathbf{b}_{21} = \hat{\mathbf{M}}^{-1}\begin{bmatrix}Z_{uu\delta u}u^2\\ M_{uu\delta u}u^2\end{bmatrix}. \tag{27}$$

## V. Simulation Results

### A. Regulation

In this section, the effectiveness of the proposed approach is demonstrated by simulating the model of REMUS AUV. The weighting matrices and gain of correction term are specified as:

$$R = 1, \tag{28}$$

$$\mathbf{Q} = I_{4\times4}, \tag{29}$$

$$\boldsymbol{\lambda} = [0.095 \quad 0 \quad 0 \quad 0]^T. \tag{30}$$

The initial condition is chosen as $\mathbf{x}(t) = [-1 \quad -0.5 \quad 0 \quad 0]^T$ and desired value is equilibrium point, zero. The AUV forward velocity constant is supposed $u = 2m/s$. The input stern plane variation bound is limited to 20 degree which keeps the performance and regulation speed of the AUV in a specific area. To improve the result in terms of the mentioned properties, the forward velocity constant and stern plane limit should be enhanced. The nominal values of the hydrodynamic parameters and the vehicle physical parameters are given in Appendix. The depth position and pitch angle of the AUV are presented in Figs. 2 and 3. Linear and angular velocities of the AUV are illustrated in Figs. 4 and 5. The control signal is shown in Fig. 6.

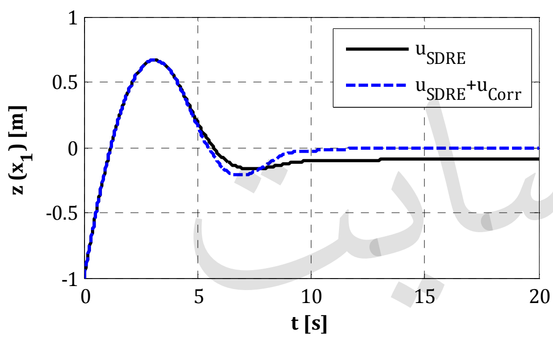


Fig. 2. The depth position of REMUS.

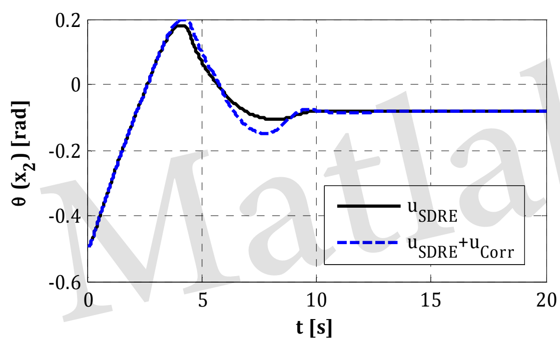


Fig. 3. The pitch angle of REMUS.

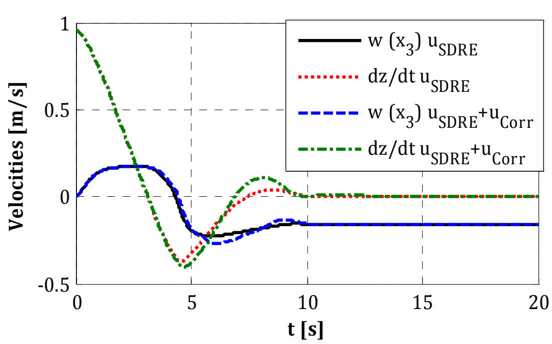


Fig. 4. The heave velocity of model.

The Fig. 2 obviously exhibited effectiveness of auxiliary sliding compensator that applied by discontinuous input term for compensation of distributed weight and buoyancy. Furthermore, it can be seen that the desired depth is achieved almost zero steady-state error with the proposed approach.

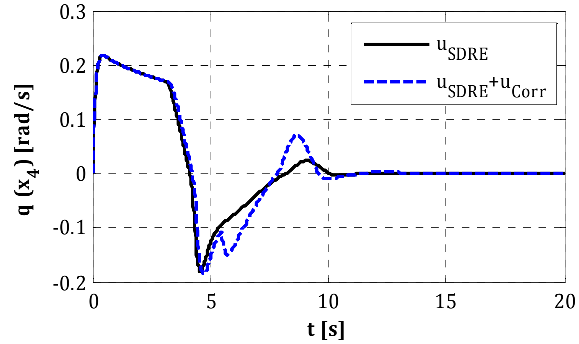


Fig. 5. The pitch angle velocity of model.

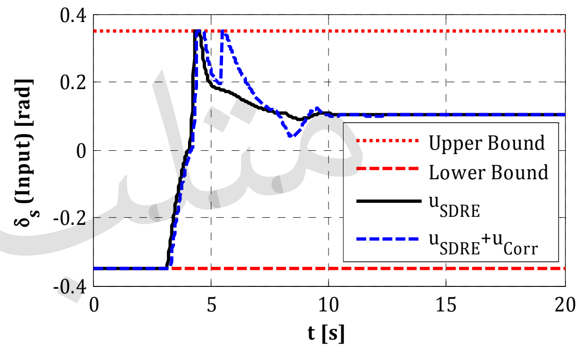


Fig. 6. Control input; stern planes angle.

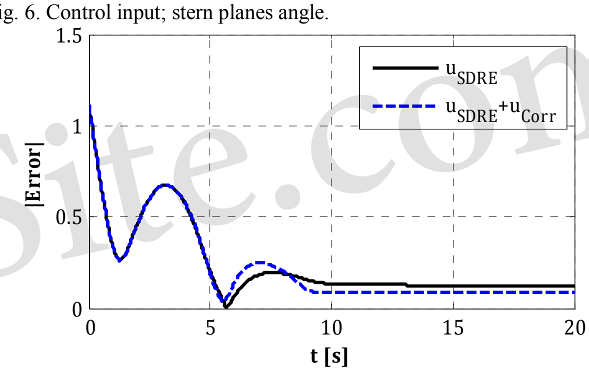


Fig. 7. Norm of states' error.

During the regulation of depth, the AUV is moving forward with a constant speed. This is the reason for not having error of second state equal to zero. This means for depth regulation or staying in a specified depth, the angle of AUV respect to $y$-axis may not be zero which steady-state angle of stern plane confirms that. So, most of the error's norm in Fig. 7 is due to second state and should not be considered as a drawback to controller design as the control of depth in $z$-axis is the main goal. As long as AUV moves with constant speed, preferable performance may not be reached. This problem can be solve with considering the position and velocity of $x_c$ direction in state vector, although it may cause the model more complex or non-affine respect to input which is suggested for future works.

## *B. Tracking*

In this part the ability of the proposed controller will be shown in tracking a predefined path. The time of simulation is regarded 20 seconds. AUV is to track a linear trajectory for its $z$ direction as $z_{des}(t)=0.1t$ which moves the AUV 2m up in depth at the end of the path. The initial condition is set as equilibrium point and similar controller's coefficients to previous section are chosen except $\boldsymbol{\lambda}=\begin{bmatrix}0.25 & 0 & 0 & 0\end{bmatrix}^T$. The trajectory tracking of first state is presented in Fig. 8. The modified controller performed its task perfectly. Figure 9 shows the variation of AUV's angle and the velocity of first and second states are shown in Figs. 10-11. The control signal is illustrated in Fig. 12 and the error of tracking in Fig. 13.

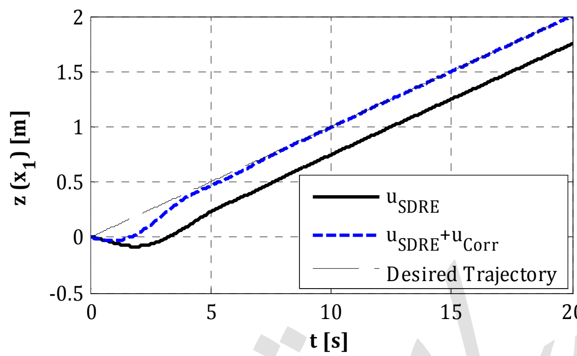


Fig. 8. The depth position of REMUS in tracking.

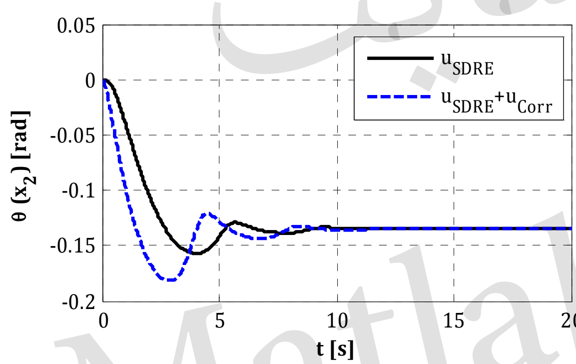


Fig. 9. The pitch angle of REMUS in tracking.

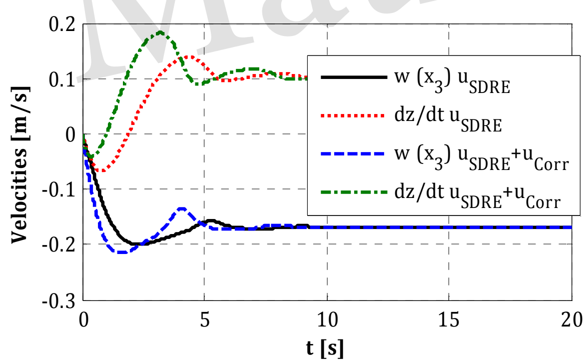


Fig. 10. The heave velocity of model in tracking.

Figure 8 shows that the correction term in modified SDRE plays a more important role in tracking problem and reduced the error near to zero while the conventional SDRE could not observe the unwanted distributed gravity/ buoyancy force (known disturbance).

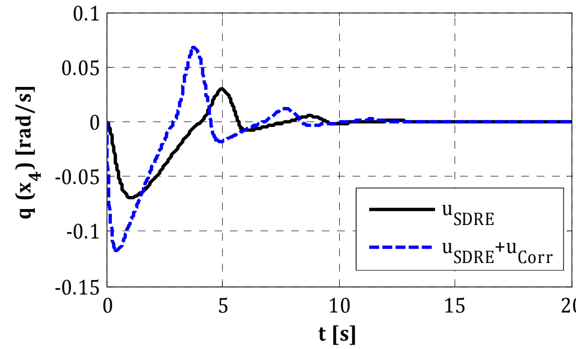


Fig. 11. The pitch angle velocity of model in tracking.

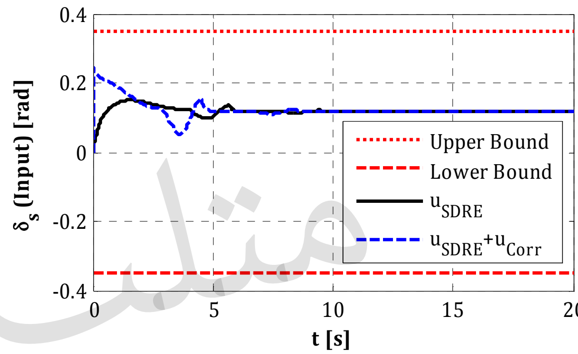


Fig. 12. Control input; stern planes in tracking.

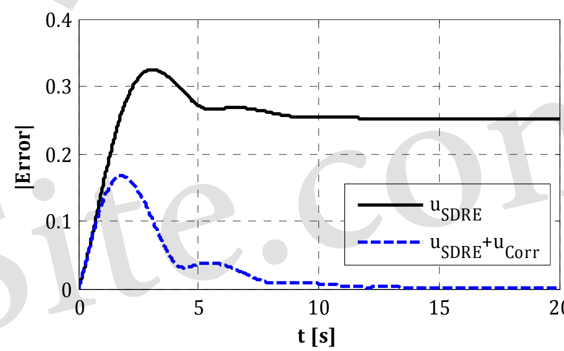


Fig. 13. Norm of states' error in tracking.

The controller is checked for $z_{des}(t)=0.5\sin(\pi/15t)$ with $\boldsymbol{\lambda}=\begin{bmatrix}0.5 & 0 & 0 & 0\end{bmatrix}^T$ as well and the trajectories and error are presented in Fig. 14 and 15.

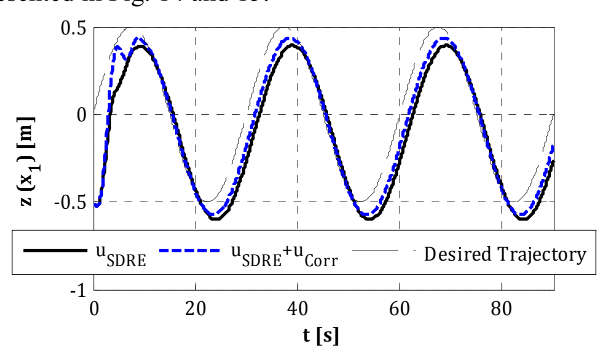


Fig. 14. The depth position of REMUS in tracking.

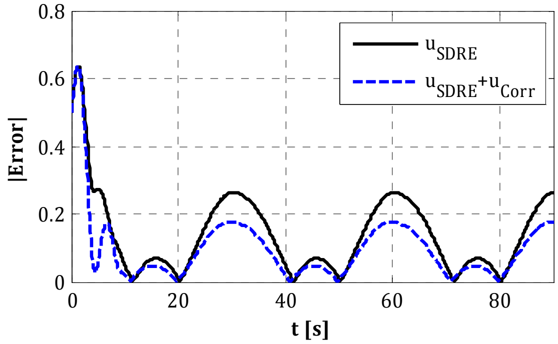


Fig. 15. Norm of states' error in tracking.

## VI. Conclusion

In this paper, the dive plane control of AUVs using the state-dependent Riccati equation method was investigated. It was assumed that the parameters of the REMUS vehicle were known precisely. Saturation bound was limited the fin angle to get the results as close as possible to practical environmental condition. A discontinuous control input term was added to conventional SDRE control law to overcome restoring force. This modified SDRE showed more robustness to unwanted distributed gravity/ buoyancy force. The modified control law with somehow robust characteristics also performed better to omit the steady state error of regulation and tracking. Simulation results were supported the capability of the modified input law in spite of the actuator limit and presence of distributed gravity/buoyancy force as a known disturbance.

## Appendix

The constant parameters of the AUV [19]:

TABLE III
The REMUS Data

| W = 299 N | B = 306 N |
|---|---|
| $I_y = 3.45 kg.m^2$ | $x_G = 0.0m$ |
| $y_G = 0.0m$ | $z_G = 0.0196m$ |
| $x_B = 0.0m$ | $y_B = 0.0m$ |
| $z_B = 0.0m$ | $m = 30.48kg$ |
| $Z_{\dot{q}} = -1.93kg/rad$ | $Z_{uw} = -28.6kg/m$ |
| $Z_{\vert w \vert w} = -131 kg.m/rad^2$ | $Z_{\dot{w}} = -35.5kg$ |
| $Z_{uu\delta} = -6.15 kg.m^{-1} rad^{-1}$ | $M_{\dot{w}} = -1.93kg.m$ |
| $M_{uu\delta} = -6.15kg/rad$ | $M_{uw} = 24kg$ |
| $M_{\dot{q}} = -4.88 kg.m^2/rad$ | $M_{\vert w \vert w} = 3.18kg$ |

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