Adaptive Position/Force Control of Robot Manipulators with Force Estimation

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***Abstract*—In this paper, we design an adaptive position/force controller for robot manipulators during constrained motion. The proposed controller can compensate for parametric uncertainties while only requiring the measurements of the position and velocity of robot arms, but not the measurements of forces at contact points. A disturbance observer is designed to estimate the constraint forces. The control method results in semiglobal asymptotic tracking performance for the joint position and bounded tracking performance for the interaction force, also the controller provides proper estimate of constraint forces. It is shown mathematically that the error of tracking desired constraint force can be decreased sufficiently by increasing the control gains. The effectiveness of the proposed method is investigated through the numerical simulation for a two-degrees-of-freedom robot manipulator acting on a horizontal worktable.**

***Keywords*—adaptive systems; position/force control; force estimation; stability analysis**

I. Introduction

In many applications of robot such as automated deburring, grinding, spot welding, scribing and precision assembly, robots are being used to perform tasks which require a contact between the robot and the workpiece. This interaction with the environment will constrain the robot motion. Consequently robotics force control is important because it not only prevents excessive contact force which could damage the workpiece, but also regulates the desired force.

So far, several force control algorithms have been proposed in the academia over the past four decades. In spite of their diversities, they can be classified into three main groups: hybrid P/F (position/force) control, hybrid impedance control, and reduced state P/F control methods.

The hybrid control approach, aims at controlling position along the unconstrained task directions and force along the constrained task directions [1],[2]. As introduced in [3],[4], the objective of an impedance controller is to maintain a desired dynamic relationship between the end-effector and the environment. These two approaches are more efficient if a precise model of the environment is available.

Reduced state P/F control method deals with the transformation of the robot dynamics into a reduced order one in which the constraint force is omitted. In [5], McClamorch and Wang developed a nonlinear transformation that amends the system dynamics into a reduced one in which the force and motion controller can be designed separately. Their controller ensures asymptotic position tracking and bounded force tracking error.

In order to compensate the system uncertainties, in [6], Carelli and Kelly presented an adaptive full state feedback controller which ensures asymptotic position tracking and bounded force tracking error. Moreover, an adaptive feedback P/F tracking controller was presented in [7]-[9].

In order to control the contact force, the force sensors data should be fed back to the controller. Conventionally, a force sensor attached to the wrist of manipulator is used to measure the contact force. However, force sensors are not popular in industrial application due to their high price. Additionally, the information of a force sensor has much noise and when the robot manipulator encounters environmental uncertainties such as high temperature and large noise, the force sensor cannot be mounted on it.

To overcome these difficulties, various force estimation methods have been proposed [10]-[13]. One approach is utilizing disturbance observer as a surrogate for force sensor. Disturbance observers consider deviations from the nominal dynamical model of the robot as an equivalent disturbance applied to the nominal model. Consequently, the output of the disturbance observer represents the external torque plus the system uncertainties. In [12], the external torque is obtained by subtracting the established modeling uncertainties from the output of disturbance observer. However, their method requires the accurate nominal model and the modeling uncertainties may engender the force estimation errors. In [10],[11], the force is estimated by considering how position estimation errors behave as a damped spring-mass system. Disturbance observers have been also used in robotic systems for hybrid P/F control where the disturbance observer works as a torque sensor [12].

It seems that the problem of force sensor free P/F controller design utilizing reduced position/force control method still remains an open forum. Considering this challenge and motivated by the concept of utilizing disturbance observer for P/F control of robotic systems, in this paper an adaptive force estimator is developed to estimate the constraint force. Afterward, this estimated force has been used to design the P/F controller without force measurements. Additionally, an adaptive parameter estimator is designed to compensate and learn the effects of the system parametric uncertainties.

The rest of the paper is organized as follows. In section II a representation of an arbitrary time varying vector in terms of Taylor series is presented and in section III, we present the constrained robot model along with some useful properties associated with the model. In section IV, the system error dynamics and a force sensor free P/F controller is introduced and the stability of the controller is analyzed by the Lyapunov theorem. In section V simulation results of the proposed controller for a two-link robot arms are given to serve as validation of the theoretical development. Finally, some concluding remarks are given in section VI.

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II. Representation of a Time Varying Function

In order to represent a time varying function, consider the following Lemma.

Lemma: Let λ_i be a time varying p-times differentiable continuous function, as shown in the Fig. 1, for a given time in the interval t_1 to t_{m+1}, $t \in [t_1, t_{m+1}]$ subdivided by m sub interval such as $[t_j, t_{j+1}]$.

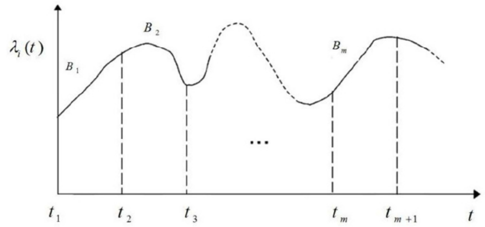

Fig. 1. Time history of an arbitrary function

The function λ_i can be represented by the following expansion

$$\lambda_i = \sum_{j=1}^{m} [1(t_j) - 1(t_{j+1})] B_j \,, \tag{1}$$

where

$$\begin{aligned} B_j &= \lambda_i(t_j) + \frac{t - t_1}{1!} \lambda_i^{(1)}(t_j) + \frac{(t - t_1)^2}{2!} \lambda_i^{(2)}(t_j) + \cdots \\ &+ \frac{(t - t_1)^{p-1}}{(p-1)!} \lambda_i^{(p-1)}(t_j) + \int_{t_j}^{t} \frac{(t - \xi)^{p-1}}{(p-1)!} \lambda_i^{(p)}(\xi) d\xi \,, \end{aligned} \tag{2}$$

and $1(t_j)$ represents the unit step function, i.e.

$$1(t_j) = \begin{cases} 1 & t \ge t_j \\ 0 & t < t_j \end{cases} \tag{3}$$

Introducing vectors

$$\mathbf{g}_j = \left(1(t_j) - 1(t_{j+1})\right) \times \left[1, \frac{t - t_j}{1!}, \ldots, \frac{(t - t_j)^{p-1}}{(p-1)!}\right], \tag{4}$$

and

$$\mathbf{a}_j = \left[\lambda_i(t_j), \lambda_i^{(1)}(t_j), \ldots, \lambda_i^{(p-1)}(t_j)\right], \tag{5}$$

then we can rewrite (1) in the form of

$$\lambda_i = \mathbf{G}_i \, \mathbf{A}_i + h_i \,, \tag{6}$$

where

$$\mathbf{G}_i = [\mathbf{g_1}, \mathbf{g_2}, \ldots, \mathbf{g_m}], \tag{7}$$

$$\mathbf{A}_i = [\mathbf{a_1}, \mathbf{a_2}, \ldots, \mathbf{a_m}]^T \,, \tag{8}$$

$$h_i = \sum_{n=1}^{m} \left([1(t_n) - 1(t_{n+1})] \times \int_{t_n}^{t} \frac{(t - \xi)^{p-1}}{(p-1)!} \lambda_i^{(p)}(\xi) d\xi \right). \tag{9}$$

Utilizing the same procedure, any time dependent $r \times 1$ vector like $\boldsymbol{\lambda}(t) = [\lambda_1(t), \lambda_2(t), \ldots, \lambda_r(t)]^T$, can be expressed by

$$\boldsymbol{\lambda}(t) = \mathbf{W}_{\boldsymbol{\lambda}} \, \boldsymbol{\psi}_{\boldsymbol{\lambda}} + \mathbf{e}_{\boldsymbol{\lambda}} \,, \tag{10}$$

where

$$\mathbf{W}_{\boldsymbol{\lambda}} = \begin{bmatrix} \mathbf{G_1} & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \mathbf{G_2} & & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \ldots & \mathbf{G_r} \end{bmatrix}, \boldsymbol{\psi}_{\boldsymbol{\lambda}} = \begin{bmatrix} \mathbf{A_1} \\ \mathbf{A_2} \\ \vdots \\ \mathbf{A_r} \end{bmatrix}, \mathbf{e}_{\boldsymbol{\lambda}} = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_r \end{bmatrix}. \tag{11}$$

If the p-th derivative of each element of vector λ such as λ_k is bounded, i.e. $\left|\lambda_k^{(p)}\right| \le \vartheta_k$, it can be deduced that

$$|h_k| \le \frac{m \, \vartheta_k \, T^{p-1}}{(p-1)!}, \tag{12}$$

where $T = \max(t_{i+1} - t_i), i = 1, \cdots, m$. Consequently

$$\|\mathbf{e}_{\boldsymbol{\lambda}}\| \le |h_1| + \ldots + |h_r| \le \rho_\lambda \,, \tag{13}$$

$$\rho_\lambda = \sum_{k=1}^{r} \frac{m T^{p-1}}{(p-1)!} \vartheta_k \,. \tag{14}$$

III. Dynamic Modeling of Robotic Systems

This section presents dynamic equations of an n-link constrained manipulator and some of their useful properties.

A. Equations of Motion

The constrained joint-space dynamics of an n-link manipulator can be described by the following Lagrangian equation [14]:

$$\boldsymbol{\tau} = \mathbf{B\Gamma} = \mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) + \mathbf{T_d} + \mathbf{f} \,, \tag{15}$$

where $\mathbf{q} \in \Re^n$, is the generalized coordinates (usually joint positions); $\boldsymbol{\tau} \in \Re^n$, is the vector of applied joints torques and forces; $\boldsymbol{\Gamma} \in \Re^n$ is the vector of actuators torques and forces; $\mathbf{B} \in \Re^{n \times n}$ is a nonsingular input matrix modeling a velocity reducers and couplings; $\mathbf{f} \in \Re^n$, is the vector of constraint forces in the joint space, which denotes the force exerted on the environment; $\mathbf{M} \in \Re^{n \times n}$, is the symmetric, bounded and P.D. (positive definite) inertia matrix; vector $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} \in \Re^n$ represents the centripetal and Coriolis torques; $\mathbf{G}(\mathbf{q}) \in \Re^n$, is the vector of gravitational torques and $\mathbf{T_d} \in \Re^n$, represents a bounded disturbance i.e.

$$\|\mathbf{T_d}\| \le \eta. \tag{16}$$

where η is a positive constant. Let us define the holonomic constraints of the system by

$$\boldsymbol{\Phi}(\mathbf{q}) = 0 \,, \tag{17}$$

where the mapping $\boldsymbol{\Phi}: \Re^n \to \Re^r$ is a twice differentiable vector function. Differentiating (17) with respect to time, we thus have

$$\mathbf{D}(\mathbf{q})\dot{\mathbf{X}} = \mathbf{0} \,, \tag{18}$$

where $\mathbf{X}, \dot{\mathbf{X}} \in \Re^n$ are, respectively, the vectors of the end effector position and velocity in the task coordinates, and $\mathbf{D}(.) \in \Re^{r \times n}$ is full rank. The constraint force $\mathbf{f}$ is then calculated by

$$\mathbf{f} = \mathbf{J}^T(\mathbf{q})\boldsymbol{\lambda} \,, \tag{19}$$

where $\boldsymbol{\lambda} \in \Re^r$ represents the generalized force multipliers associated with the constraints, generally a time varying vector and $\mathbf{J}$ is calculated by $\mathbf{J} = \mathbf{D}\mathbf{J_e}$, where $\mathbf{D}(.)$ is defined by (18) and $\mathbf{J_e} \in \Re^{n \times n}$ denotes the relating joint space and task space Jacobian matrix.

Defining $\mathbf{x_1} = \mathbf{q}$ and $\mathbf{x_2} = \dot{\mathbf{q}}$, (15) can be written in the state form as

$$\begin{cases} \dot{\mathbf{x}}_\mathbf{1} = \mathbf{x_2} \,, \\ \mathbf{M}\dot{\mathbf{x}}_\mathbf{2} = -\mathbf{C}(\mathbf{x_1}, \mathbf{x_2})\mathbf{x_2} - \mathbf{G}(\mathbf{x_1}) - \mathbf{T_d} - (\mathbf{I} + \mathbf{K_f})\mathbf{f} + \mathbf{K_f}\mathbf{f} + \boldsymbol{\tau} \,, \end{cases} \tag{20}$$

where $\mathbf{K_f}$ is an $n\times n$ selective diagonal P.D. matrix.

B. Properties of Equations of Motion

The constrained robot system has the following properties.

Property 1: The inertia matrix $\mathbf{M(q)}$ is known to be P.D. and symmetric.

Property 2: A suitable definition of $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})$ makes the matrix $\frac{1}{2}\dot{\mathbf{M}}-\mathbf{C}$ skew symmetric. So (21) holds for any arbitrary vector $\xi\in\Re^n$,

$$\xi^T[\frac{1}{2}\dot{\mathbf{M}}-\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})]\,\xi=0. \quad (21)$$

Property 3: An upper bound can be placed on the norm of Coriolis and centripetal matrix, i.e.

$$\|\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\|\le K_c\|\dot{\mathbf{q}}\|, \quad (22)$$

where K_c is a positive constant.

Definition: For any arbitrary P.D. or N.D. (negative definite) symmetric matrix such as $\mathbf{L}$, throughout this paper by L_m and L_M, the authors mean the minimum and maximum eigenvalues of that matrix. Hence, for any arbitrary vector ξ we can state

$$L_m\|\xi\|^2\le\xi^{\mathbf{T}}\mathbf{L}\xi\le L_M\|\xi\|^2. \quad (23)$$

Note that $\|.\|$ represents the 2 induced matrix norm for matrices and Euclidean norm for vectors.

IV. Adaptive Controller Design

In this section, the main result is presented in the form of a theorem and the stability of the proposed force sensor free P/F controller in desired position and force tracking and parameter and force estimation are evaluated by the Lyapunov theorem.

A. Error Dynamics

Let us define

$$\bar{\mathbf{x}}_1=\mathbf{q}-\mathbf{q_d},\ \bar{\mathbf{x}}_2=\dot{\mathbf{q}}-\dot{\mathbf{q}}_\mathbf{d}+\mathbf{P}(\mathbf{q}-\mathbf{q_d}), \quad (24)$$

where $\mathbf{P}$ is an $n\times n$ symmetric P.D. matrix and $\mathbf{q_d}$ and $\dot{\mathbf{q}}_\mathbf{d}$ represent individually the desired position and velocity vectors. Hence, (20) can be expressed by

$$\begin{cases}\dot{\bar{\mathbf{x}}}_1=\bar{\mathbf{x}}_2-\mathbf{P}\bar{\mathbf{x}}_1,\\ \mathbf{M}\dot{\bar{\mathbf{x}}}_2=-\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\bar{\mathbf{x}}_2-\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})(-\mathbf{P}\bar{\mathbf{x}}_1+\dot{\mathbf{q}}_\mathbf{d})-\mathbf{G}(\mathbf{q})+\boldsymbol{\tau}\\ \qquad-(\mathbf{I}+\mathbf{K_f})\mathbf{f}+\mathbf{K_f}\mathbf{f}-\mathbf{T_d}-\mathbf{M}\ddot{\mathbf{q}}_\mathbf{d}+\mathbf{MP}(\bar{\mathbf{x}}_2-\mathbf{P}\bar{\mathbf{x}}_1).\end{cases} \quad (25)$$

Let us define

$$\mathbf{Y}_1(\mathbf{q},\dot{\mathbf{q}},t)\boldsymbol{\theta}_\mathbf{0}=\mathbf{MP}(\bar{\mathbf{x}}_2-\mathbf{P}\bar{\mathbf{x}}_1)-\mathbf{M}\ddot{\mathbf{q}}_\mathbf{d}-\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})(-\mathbf{P}\bar{\mathbf{x}}_1+\dot{\mathbf{q}}_\mathbf{d})-\mathbf{G}(\mathbf{q}), \quad (26)$$

and

$$\mathbf{Y}_2(\mathbf{q})\boldsymbol{\lambda}(t)=\mathbf{K_f}\mathbf{f}, \quad (27)$$

where $\mathbf{Y_1}(\mathbf{q},\dot{\mathbf{q}},t)\in\Re^{n\times l}$ and $\mathbf{Y_2}(\mathbf{q})=\mathbf{K_f}\mathbf{J}^T(\mathbf{q})\in\Re^{n\times r}$ are regressor matrices and the vector $\boldsymbol{\theta}_\mathbf{0}\in\Re^l$ represents the system constant parameters and $\boldsymbol{\lambda}(t)\in\Re^r$ is the vector of unmeasured Lagrangian multipliers. Consequently, we can rewrite (25) as

$$\begin{cases}\dot{\bar{\mathbf{x}}}_1=\bar{\mathbf{x}}_2-\mathbf{P}\bar{\mathbf{x}}_1,\\ \mathbf{M}\dot{\bar{\mathbf{x}}}_2=\mathbf{Y}_1(\mathbf{q},\dot{\mathbf{q}},t)\boldsymbol{\theta}_\mathbf{0}+\mathbf{Y}_2(\mathbf{q})\boldsymbol{\lambda}(t)-(\mathbf{I}+\mathbf{k_f})\mathbf{f}-\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\bar{\mathbf{x}}_2-\mathbf{T_d}+\boldsymbol{\tau}.\end{cases} \quad (28)$$

Since the function $\boldsymbol{\lambda}(t)$ is piecewise continuous, we can expand $\boldsymbol{\lambda}(t)$ with the notation defined in section II (equation (10)), to obtain

$$\begin{cases}\dot{\bar{\mathbf{x}}}_1=\bar{\mathbf{x}}_2-\mathbf{P}\bar{\mathbf{x}}_1,\\ \mathbf{M}\dot{\bar{\mathbf{x}}}_2=\mathbf{Y}_1(\mathbf{q},\dot{\mathbf{q}},t)\boldsymbol{\theta}_\mathbf{0}+\mathbf{Y}_3(\mathbf{q},\dot{\mathbf{q}},t)\boldsymbol{\psi}_\lambda(t)+\mathbf{Y}_2(\mathbf{q},\dot{\mathbf{q}},t)\mathbf{e}_\lambda\\ \qquad-(\mathbf{I}+\mathbf{k_f})\mathbf{f}-\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\bar{\mathbf{x}}_2-\mathbf{T_d}+\boldsymbol{\tau},\end{cases} \quad (29)$$

where $\mathbf{Y_3}=\mathbf{Y_2}\,\mathbf{W}_\lambda$.

B. Adaptive Force Sensor Free P/F Controller

In this section, assuming the system parameters ($\boldsymbol{\theta}_\mathbf{0}$) are uncertain, a force sensor free adaptive P/F controller is designed. The controller deals with the constraint force estimation as well as the system parametric uncertainties. The controller ensures asymptotic convergence of position and velocity errors to their desired trajectory and at the same time keeps the force estimation/tracking errors bounded.

In addition to adaptation law of unknown parameters in force estimation, a conventional adaptation law should be implemented to compensate the effect of system parametric uncertainties defined by $\boldsymbol{\theta}_\mathbf{0}$. The following controller is proposed.

$$\boldsymbol{\tau}=-\mathbf{Y}_1\hat{\boldsymbol{\theta}}_\mathbf{0}-\mathbf{Y}_3\hat{\boldsymbol{\psi}}_\lambda-\mathbf{K_v}\bar{\mathbf{x}}_2+(\mathbf{I}+\mathbf{K_f})\mathbf{f_d}+\Delta\mathbf{u}, \quad (30)$$

$$\dot{\hat{\boldsymbol{\theta}}}_\mathbf{0}=\boldsymbol{\Gamma}_\mathbf{1}^{-1}\mathbf{Y}_\mathbf{1}^T\bar{\mathbf{x}}_2, \quad (31)$$

where $\mathbf{K_v}\in\Re^{n\times n}$ and $\boldsymbol{\Gamma}_1$ are P.D. gain matrices and $\mathbf{f_d}=\mathbf{J}^T(\mathbf{q})\boldsymbol{\lambda}_\mathbf{d}$, represents the desired force vector. $\Delta\mathbf{u}$ is a part of controller defined by

$$\Delta\mathbf{u}=-\frac{\rho^2\bar{\mathbf{x}}_2}{\rho\|\bar{\mathbf{x}}_2\|+\varepsilon^2}. \quad (32)$$

$\hat{\boldsymbol{\psi}}_\lambda$ is the coefficient of estimated force ruled by

$$\dot{\hat{\boldsymbol{\psi}}}_\lambda=\boldsymbol{\Gamma}_\mathbf{3}^{-1}\mathbf{Y}_\mathbf{3}^T\bar{\mathbf{x}}_2, \quad (33)$$

where $\boldsymbol{\Gamma}_\mathbf{3}$ is a design P.D. matrix.

In (32), ρ is a positive function calculated by

$$\rho=\rho_\lambda\|\mathbf{Y}_2(.)\|+\rho_f+\eta, \quad (34)$$

where ρ_λ and η are positive constants individually defined in (14) and (16). ρ_f is a positive constant which should be designed properly, it will be discussed later on. Also, ε in (32) is a decreasing positive function calculated by

$$\dot{\varepsilon}=-k_\varepsilon^{-1}\varepsilon, \quad (35)$$

where k_ε is positive constant and $\varepsilon(0)>0$.

The closed loop error dynamics, can be written as

$$\begin{cases}\dot{\bar{\mathbf{x}}}_1=\bar{\mathbf{x}}_2-\mathbf{P}\bar{\mathbf{x}}_1,\\ \mathbf{M}\dot{\bar{\mathbf{x}}}_2=\mathbf{Y}_1\tilde{\boldsymbol{\theta}}_\mathbf{0}-(\mathbf{I}+\mathbf{K_f})\tilde{\mathbf{f}}+\mathbf{Y}_3\tilde{\boldsymbol{\psi}}_\lambda(t)+\mathbf{Y}_2\mathbf{e}_\lambda-\mathbf{K_v}\bar{\mathbf{x}}_2-\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\bar{\mathbf{x}}_2-\mathbf{T_d}+\Delta\mathbf{u},\end{cases} \quad (36)$$

where $\tilde{\boldsymbol{\theta}}_\mathbf{0}=\boldsymbol{\theta}_\mathbf{0}-\hat{\boldsymbol{\theta}}_\mathbf{0}$, $\tilde{\boldsymbol{\psi}}_\lambda=\boldsymbol{\psi}_\lambda-\hat{\boldsymbol{\psi}}_\lambda$ and $\tilde{\mathbf{f}}=\mathbf{f}-\mathbf{f_d}$.

The stability of the system will be analyzed in the next subsection.

Remark 1: The desired constraint force $\mathbf{f_d}$ is related to the desired force variables $\boldsymbol{\lambda}_\mathbf{d}$ and the desired sensor forces $\mathbf{F_{sd}}$ by

$$\mathbf{f_d}=\mathbf{J}^T(\mathbf{q})\boldsymbol{\lambda}_\mathbf{d}=\mathbf{J_s}^T(\mathbf{q})\mathbf{F_{sd}}, \quad (37)$$

where $\mathbf{J_s}$ is an $n \times n$ Jacobian sensor matrix, and $\mathbf{F_{sd}}$ is an $n \times 1$ vector of desired sensor forces represented in a Cartesian coordinate.

C. Stability Analysis

The stability and performance of the control system in the trajectory tracking and force control are analyzed by the following theorem. The theorem is developed based on the Lyapunov arguments where a P.D. Lyapunov function is shown to have a N.D. time derivative, once it starts from a bounded initial state vector $\boldsymbol{\gamma} = [\bar{\mathbf{x}}_1^T, \bar{\mathbf{x}}_2^T, \tilde{\boldsymbol{\theta}}_0^T, \tilde{\boldsymbol{\psi}}_\lambda^T]^T$.

Before presenting the nucleus theorem let us introduce two individual sets Ω_1 and $\Omega_2\,(\Omega_1 \subset \Omega_2)$ defined by

$$\Omega_1 = \left\{ \|\mathbf{E}(t)\|^2 \le \frac{N_m}{N_M}\ell \;\; \& \; \|\tilde{\mathbf{f}}(t)\| \le \frac{1}{\|\mathbf{I}+\mathbf{K_f}\|}\rho_f \right\}, \qquad (38)$$

$$\Omega_2 = \left\{ \|\mathbf{E}(t)\|^2 \le \ell \;\; \& \;\; \|\tilde{\mathbf{f}}(t)\| \le \frac{1}{\|\mathbf{I}+\mathbf{K_f}\|}\rho_f \right\}, \qquad (39)$$

where $\mathbf{E}(t) = [\boldsymbol{\gamma}^T, \varepsilon]^T$ is an augmented state vector and N_m and N_M represent the minimum and the maximum eigenvalues of the augmented matrix $\mathbf{N}$, defined by

$$\mathbf{N} = diag\left\{\mathbf{I}, \mathbf{M}, \boldsymbol{\Gamma}_1, \boldsymbol{\Gamma}_3, K_\varepsilon^{-1}\right\}. \qquad (40)$$

Positive constants ℓ and ρ_f, define the boundary of these two sets.

For all the trajectories in Ω_2, the following bounds can be assumed, i.e. $\|\mathbf{Y_2}\| \le y_2$, $\max_{\forall\, \mathbf{E}\in\Omega_2}\|\mathbf{Y}_1\| = y_1$ and $\max_{\forall\, \mathbf{E}\in\Omega_2}\|\mathbf{Y_3}\| = y_3$.

Theorem 1: The input controller calculated by (30) results in semiglobal asymptotic stability of position and velocity tracking error and bounded force tracking error in the system (15), i.e.

$$\lim_{t\to\infty}\mathbf{q} = \mathbf{q_d},\; \lim_{t\to\infty}\dot{\mathbf{q}} = \dot{\mathbf{q}}_\mathbf{d}, \qquad (41)$$

$$\forall t \ge 0 \quad \|\tilde{\mathbf{f}}(t)\| \le \frac{1}{\|\mathbf{I}+\mathbf{K_f}\|}\rho_f, \qquad (42)$$

provided that:

1) The desired trajectories being bounded and moreover the system augmented error initialized in Ω_1 or simply, $\mathbf{E}(0)$ belongs to set Ω_1. ℓ and ρ_f are positive constants represent the bounds of Ω_1, Ω_2 and should satisfy the following conditions.

$$\ell \le \frac{1}{\mu^2} \times [\rho_f - (M_M\alpha + y_2\rho_\lambda + \eta + \rho)]^2, \qquad (43)$$

$$\rho_f \ge M_M\alpha + y_2\rho_\lambda + \eta + \rho, \qquad (44)$$

with

$$\mu = y_1 + y_3 + K_{vM} + K_c(\ell + P_M\ell + \|\dot{\mathbf{q}}_\mathbf{d}\|\sqrt{\ell}). \qquad (45)$$

2) The controller gains satisfy the following conditions

$$K_{vm} > \frac{1}{2}, P_m > \frac{1}{2}. \qquad (46)$$

Proof: Consider a so called P.D. Lyapunov function calculated by

$$V(\mathbf{E}) = \frac{1}{2}\mathbf{E}^T\mathbf{N}\mathbf{E}. \qquad (47)$$

Differentiating (47) with respect to time, we thus obtain

$$\begin{aligned}\dot{V} =& -\bar{\mathbf{x}}_1^T\mathbf{P}\bar{\mathbf{x}}_1 + \bar{\mathbf{x}}_1^T\bar{\mathbf{x}}_2 + \bar{\mathbf{x}}_2^T[\tfrac{1}{2}\dot{\mathbf{M}} - \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})]\bar{\mathbf{x}}_2 \\ &+\bar{\mathbf{x}}_2^T[\mathbf{Y}_1\tilde{\boldsymbol{\theta}}_0 - (\mathbf{I}+\mathbf{K_f})\tilde{\mathbf{f}} + \mathbf{Y_3}\tilde{\boldsymbol{\psi}}_\lambda(t) + \mathbf{Y_2}\mathbf{e}_\lambda - \mathbf{K_v}\bar{\mathbf{x}}_2 - \frac{\rho^2\bar{\mathbf{x}}_2}{\rho\|\bar{\mathbf{x}}_2\| + \varepsilon^2} - \mathbf{T_d}] \\ &-\tilde{\boldsymbol{\theta}}_0^T(\mathbf{Y}_1^T\bar{\mathbf{x}}_2) - \tilde{\boldsymbol{\psi}}_\lambda^T(\mathbf{Y}_3^T\bar{\mathbf{x}}_2) - \varepsilon^2.\end{aligned} \qquad (48)$$

Simplifying (48) yields to

$$\begin{aligned}\dot{V} \le& (\tfrac{1}{2} - P_m)\|\bar{\mathbf{x}}_1\|^2 + \|\mathbf{I}+\mathbf{K_f}\|\|\bar{\mathbf{x}}_2\|\|\tilde{\mathbf{f}}\| + (\tfrac{1}{2} - K_{v_m})\|\bar{\mathbf{x}}_2\|^2 \\ &+ (\rho_\lambda\|\mathbf{Y_2}(.)\| + \eta)\|\bar{\mathbf{x}}_2\| - \frac{\rho^2\|\bar{\mathbf{x}}_2\|^2}{\rho\|\bar{\mathbf{x}}_2\| + \varepsilon^2} - \varepsilon^2.\end{aligned} \qquad (49)$$

For any trajectory in the region Ω_2, (49) can be simplified to

$$\dot{V} \le (\tfrac{1}{2} - P_m)\|\bar{\mathbf{x}}_1\|^2 + (\tfrac{1}{2} - K_{v_m})\|\bar{\mathbf{x}}_2\|^2 + \rho\|\bar{\mathbf{x}}_2\| - \frac{\rho^2\|\bar{\mathbf{x}}_2\|^2}{\rho\|\bar{\mathbf{x}}_2\| + \varepsilon^2} - \varepsilon^2, \qquad (50)$$

where ρ is defined in (34). If K_{v_m} and P_m are selected greater than one half unit, one can simplify (50) to obtain

$$\dot{V} \le -\varepsilon_1\|\bar{\mathbf{x}}_1\|^2 - \varepsilon_2\|\bar{\mathbf{x}}_2\|^2 - \frac{\varepsilon^4}{\rho\|\bar{\mathbf{x}}_2\| + \varepsilon^2}, \qquad (51)$$

where ε_1 and ε_2 are positive constants.

Integrating (51) with respect to time yields to

$$V(t) \le V(0) - \int_0^t \left(\varepsilon_1\|\bar{\mathbf{x}}_1\|^2 + \varepsilon_2\|\bar{\mathbf{x}}_2\|^2\right)dt. \qquad (52)$$

Considering the Rayleigh theorem and using the result in (52), one obtains

$$N_m\|\mathbf{E}(t)\|^2 \le V(t) \le N_M\|\mathbf{E}(0)\|^2 - \int_0^t \left(\varepsilon_1\|\bar{\mathbf{x}}_1\| + \varepsilon_2\|\bar{\mathbf{x}}_2\|\right)dt. \qquad (53)$$

Now if we consider the definition of Ω_1, it is clear that for any trajectory initialized in Ω_1, we will have

$$\forall t \ge 0 \quad \|\mathbf{E}(t)\|^2 \le \ell. \qquad (54)$$

In the Appendix, it is shown that in any region that (54) is satisfied, the vector $\dot{\bar{\mathbf{x}}}_2$ will be a bounded vector, i.e.

$$\|\dot{\bar{\mathbf{x}}}_2\| \le \alpha. \qquad (55)$$

Through the substitutions of (54) and (55) in (36), it can be inferred that

$$\|\tilde{\mathbf{f}}(t)\| \le \frac{1}{\|\mathbf{I}+\mathbf{K_f}\|}[M_M\alpha + y_2\rho_\lambda + \rho + \eta + \mu\sqrt{\ell}]. \qquad (56)$$

Therefore by a proper definition of ℓ in (43), we obtain

$$\|\tilde{\mathbf{f}}(t)\| \le \frac{1}{\|\mathbf{I}+\mathbf{K_f}\|}\rho_f. \qquad (57)$$

Consequently it can be concluded that any trajectory initialized in Ω_1 will remain in Ω_2. Moreover, from (53), it can be concluded that

$$\lim_{t\to\infty}\|\mathbf{E}(t)\|^2 \le \ell - \frac{1}{N_m}\int_0^\infty \left(\varepsilon_1\|\bar{\mathbf{x}}_1\|^2 + \varepsilon_2\|\bar{\mathbf{x}}_2\|^2\right)dt. \qquad (58)$$

Hence it can be stated that $\lim_{t\to\infty}\bar{\mathbf{x}}_1 = \mathbf{0}, \lim_{t\to\infty}\bar{\mathbf{x}}_2 = \mathbf{0}$, therefore

$$\lim_{t\to\infty}\mathbf{q} = \mathbf{q_d} \;\&\; \lim_{t\to\infty}\dot{\mathbf{q}} = \dot{\mathbf{q}}_\mathbf{d}. \qquad (59)$$

In other words, the position and velocity tracking errors are asymptotically stable.

Remark 2: From (57) and (59) it can be ensured that for any trajectory initialized in Ω_1, the position and velocity tracking errors will asymptotically converge to zero and the desired force tracking will stay bounded. Moreover, through a proper design of matrix $\mathbf{K_f}$ it can be deduced that, the bound of desired force tracking can be decreased considerably.

V. Illustrative Example

In order to verify the performance of the controller, in this section we used the proposed method to simultaneously control the joint position and the constraint force of a two link serial manipulator which works out of gravity and restricted to move on a frictionless horizontal manifold.

The schematic of the system is shown in Fig. 2 and the system parameters are considered as $m_1 = m_2 = 1[kg]$, $L_1 = L_2 = 1,\ L_{c_1} = L_{c_2} = .5[m]$, where m_i, L_i, L_{ci} represent individually the links mass, the links length and the position of the center of mass of links. The mass of links is assumed to be constant but uncertain. The dynamic of the system is calculated by (15) with

$$\mathbf{M} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}, \mathbf{C} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}, \mathbf{G} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \mathbf{T_d} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \mathbf{B} = \mathbf{I}_{2\times 2},$$
$$M_{11} = m_1 l_{c_1}{}^2 + I_1 + m_2(l_1{}^2 + l_{c_2}{}^2 + 2 l_1 l_{c_2} \cos q_2) + I_2, \tag{60}$$
$$M_{12} = M_{21} = m_2 l_1 l_{c_2} \cos q_2 + m_2 l_{c_2}{}^2 + I_2, M_{22} = m_2 l_{c_2}{}^2 + I_2,$$
$$C_{11} = -m_2 l_1 l_{c_2} \sin q_2 \dot{q}_2, C_{12} = -m_2 l_1 l_{c_2} \sin q_2 (\dot{q}_2 + \dot{q}_1),$$
$$C_{21} = m_2 l_1 l_{c_2} \sin q_2 \dot{q}_1, C_{22} = 0,$$

As mentioned before, we simultaneously control the joint position and constraint force of the manipulator. For the simulation purpose, the desired joint position trajectory for the first link is considered as $q_{1d} = \sin t$.

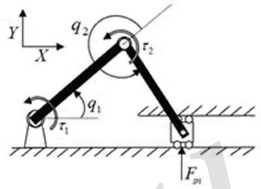

Fig. 2. Schematic of constrained two link manipulator

Since the system is constrained to move on a horizontal surface, the joint trajectory of link 2 is a function of the joint trajectory of link 1 and can be computed from the following constraint equation

$$l_1 \sin(q_1) + l_2 \sin(q_1 + q_2) = 0. \tag{61}$$

The system is constrained to move on a frictionless horizontal surface as a consequence, the desired sensor force in the X direction is equal to zero. For the simulation purpose the desired force is considered as

$$\boldsymbol{\lambda}_{\mathbf{d}} = 6, \mathbf{F_{sd}} = \begin{bmatrix} 0 & 6 \end{bmatrix}^T. \tag{62}$$

The constrained and sensor Jacobian matrices are individually defined by

$$\mathbf{J_s} = \mathbf{J}_e = \begin{bmatrix} -l_1 \sin(q_1) - l_2 \sin(q_1 + q_2) & -l_2 \sin(q_1 + q_2) \\ l_1 \cos(q_1) + l_2 \cos(q_1 + q_2) & l_2 \cos(q_1 + q_2) \end{bmatrix}, \tag{63}$$
$$\mathbf{D} = [0 \quad 1], \mathbf{J} = \begin{bmatrix} l_1 \cos(q_1) + l_2 \cos(q_1 + q_2) & l_2 \cos(q_1 + q_2) \end{bmatrix},$$

The initial conditions for independent coordinate joint position and velocities are $q_1(0) = .1(rad)$ and $\dot{q}_1(0) = 0(^{rad}/_{\sec})$ and the controller gains are set to be

$$\mathbf{K_v} = diag\{30,30\}, \mathbf{P} = diag\{3,3\}, K_\varepsilon = 1, \varepsilon(0) = 100. \tag{64}$$

The control gains are also set as $\mathbf{\Gamma_1} = 10, \mathbf{\Gamma_3} = 10, \mathbf{K_f} = 1$.

In order to expand the generalized force multipliers $\boldsymbol{\lambda}$ by the Taylor formula (see section II, especially Fig. (1)), equal time intervals are considered, i.e. $T = t_{s+1} - t_s = .4 \quad \forall s = 1, \ldots\ldots, m$.

The simulation results are presented in Figs. 3-8. The angular position tracking error ($\bar{\mathbf{x}}_1$), is represented in Fig. 3. The joint velocity tracking error ($\dot{\bar{\mathbf{x}}}_1$), is shown in Fig. 4. Fig. 5 represents the time history of the contact force from which it can be observed that the absolute value of the force tracking error is bounded. Figs. 6 and 7 depict the time history of the constraint force estimation error and the estimation of the link masses respectively. Time history of the applied torque is depicted in Fig. 8.

Since the desired trajectory is persistently exciting, hence the estimations converge to the vicinity of the actual parameters.

VI. Conclusion

In this paper we consider the problem of adaptive P/F control of robotic systems in the absence of force sensor. The constraint force is expanded by the well known Taylor series and a novel adaptive update law is introduced to estimate the unknown coefficients of the Taylor series. An adaptive P/F controller is designed by the sole sensory feedback of the robot joint position and velocities. The method ensures asymptotic convergence of the trajectory tracking erros and boundedness of the desired force tracking error. By implementing the method for a two-link manipulator numerically, it has been seen that the robot is able to track simultaneously a desired trajectory and a desired constraint force while the uncertain parameters estimation and the estimation of the contact force are updated simultaneously. Numerical results illustrate a satisfactory performance of the proposed controller, both in desired trajectory tracking and keeping the constraint force in a specific bound.

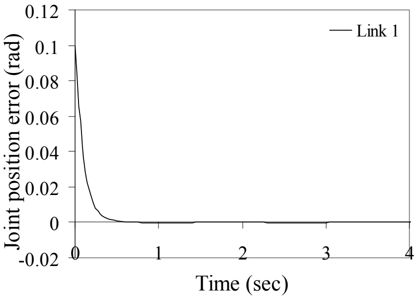

Fig. 3. Time history of the joint position error

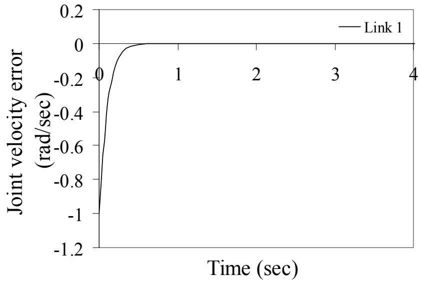

Fig. 4. Time history of the joint velocity error

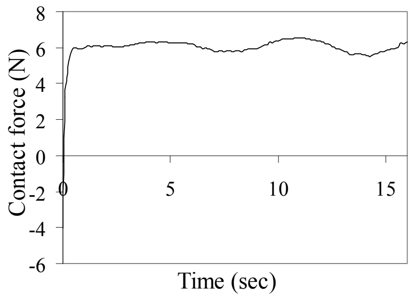

Fig. 5. Time history of the normal contact force

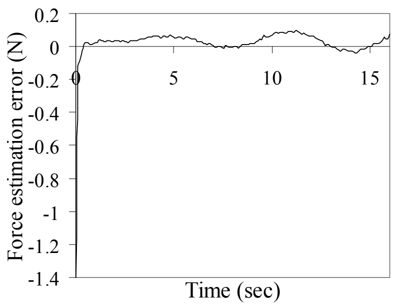

Fig. 6. Time history of the force estimation error

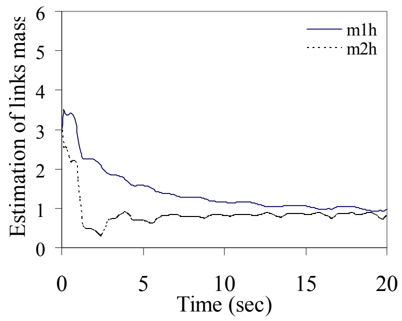

Fig. 7. Time history of the links mass estimation

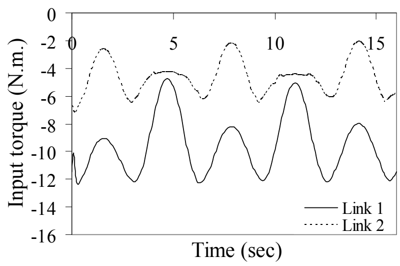

Fig. 8. Time history of input torques

APPENDIX

Since the presence of r constraints causes the manipulator to lose r degrees of freedom, the manipulator is left with only $n-r$ degrees of freedom. In this case, $n-r$ linear independent coordinates are sufficient to characterize the constrained motion. Hence, through choosing $n-r$ linear independent coordinate of n-joint variables denoted by $\mathbf{q}^1=[q_1{}^1,...,q_{n-r}{}^1]$. The constraint equation (17) can be simplified to $\mathbf{q}^2=\boldsymbol{\sigma}(\mathbf{q}^1)$.

Let's define

$$\mathbf{L}=\begin{bmatrix}\mathbf{I}_{n-r} & (\dfrac{\partial\boldsymbol{\sigma}}{\partial\mathbf{q}^1})^T\end{bmatrix}^T, \tag{65}$$

hence (66) holds

$$\mathbf{J}(\mathbf{q}^1)\mathbf{L}(\mathbf{q}^1)=\mathbf{L}^T(\mathbf{q}^1)\mathbf{J}^T(\mathbf{q}^1)=0. \tag{66}$$

Equation (36) can be written in the reduced form [9]:

$$\begin{cases}\dot{\bar{\mathbf{x}}}_1=\bar{\mathbf{x}}_2-\mathbf{P}\bar{\mathbf{x}}_1\\ \mathbf{M}^1\ddot{\mathbf{q}}^1=\mathbf{L}^T\mathbf{M}(\ddot{\mathbf{q}}_d+\mathbf{P}\dot{\mathbf{q}}_d)-\mathbf{L}^T\mathbf{M}\dot{\mathbf{L}}\dot{\mathbf{q}}^1-\mathbf{L}^T\mathbf{MPL}\dot{\mathbf{q}}^1+\mathbf{L}^T\mathbf{Y}_1\tilde{\boldsymbol{\theta}}_0\\ \quad+\mathbf{L}^T\mathbf{Y}_3\tilde{\boldsymbol{\psi}}_\lambda(t)-\mathbf{L}^T\mathbf{K}_v\bar{\mathbf{x}}_2+\mathbf{L}^T\mathbf{Y}_2\mathbf{e}_\lambda-\mathbf{L}^T\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\bar{\mathbf{x}}_2-\mathbf{L}^T\mathbf{T}_d+\mathbf{L}^T\Delta\mathbf{u},\end{cases} \tag{67}$$

where the P.D. matrix $\mathbf{M}^1$ is calculated by $\mathbf{M}^1=\mathbf{L}^T\mathbf{ML}$.

Since the desired trajectories and their first and second derivatives are assumed to be upper bounded, and matrices $\mathbf{L}$ and $\dot{\mathbf{L}}$ are upper bounded for bounded states, hence one may conclude that in any region that (51) is satisfied, $\ddot{\mathbf{q}}^1$ is a bounded vector and so are $\ddot{\mathbf{q}}$ and $\dot{\bar{\mathbf{x}}}_2$, i.e. $\|\dot{\bar{\mathbf{x}}}_2\|\le\alpha$.

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